



Chapter 5 Discounted Cash Flow Valuation

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

Key Concepts and Skills

After studying this chapter, you should be able to:

- Determine the future value and present value of investments with multiple cash flows.
- Calculate loan payments, and find the interest rate on a loan.
- Describe how loans are amortized or paid off.
- Explain how interest rates are quoted (and misquoted).

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-2

Chapter Outline

- 5.1 Future and Present Values of Multiple Cash Flows
- 5.2 Valuing Level Cash Flows: Annuities and Perpetuities
- 5.3 Comparing Rates: The Effect of Compounding Periods
- 5.4 Loan Types and Loan Amortization

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-3



Multiple Cash Flows Computational Methods

- TVM Formulas
- Texas Instruments BA II+
 - PV/FV keys
 - Cash Flow Worksheet
 - Present Value only
- Excel Spreadsheet/Functions

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-4



Future Value: Multiple Cash Flows

Example 5.1

- You think you will be able to deposit \$4,000 at the end of each of the next three years in a bank account paying 8 percent interest.
- You currently have \$7,000 in the account.
- How much will you have in 3 years?
- How much in 4 years?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-5

Future Value: Multiple Cash Flows

Example 5.1 - Formulas

- Find the value at year 3 of each cash flow and add them together.
 - Year 0: $FV = \$7,000(1.08)^3 = \$ 8,817.98$
 - Year 1: $FV = \$4,000(1.08)^2 = \$ 4,665.60$
 - Year 2: $FV = \$4,000(1.08)^1 = \$ 4,320.00$
 - Year 3: value = \$ 4,000.00
 - Total value in 3 years = \$21,803.58
- Value at year 4 = $\$21,803.58(1.08) = \$23,547.87$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-6

Future Value: Multiple Cash Flows

Example 5.2

- If you deposit \$100 in one year, \$200 in two years and \$300 in three years.
- How much will you have in three years at 7 percent interest?
- How much in five years if you don't add additional amounts?
 - Year 1 CF: 2 N; -100 PV; 7 I/Y; CPT FV = 114.49
 - Year 2 CF: 1 N; -200 PV; 7 I/Y; CPT FV = 214.00
 - Year 3 CF: 0 N; -300 PV; 7 I/Y; CPT FV = 300.00
 - Total FV₃ = 628.49
 - Total FV₅ = 628.49 * (1.07)² = 719.56

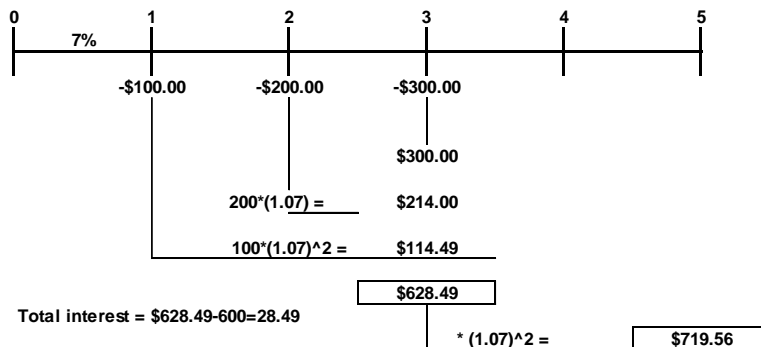
Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-7

Future Value: Multiple Uneven Cash Flows

Example 5.2 – Formulas & Time Line

TIMELINE



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-8

Future Value: Multiple Cash Flows Example 5.2

Rate	7%			
Year	Nper	CF	FV	Function
1	2	-100	\$114.49	=FV(0.07,2,0,-100)
2	1	-200	\$214.00	=FV(0.07,1,0,-200)
3	0	-300	\$300.00	=FV(0.07,0,0,-300)
Total FV at Year 3			\$628.49	
Total FV at Year 5			\$719.56	=(628.49)*(1.07)^2



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-9

Future Value: Multiple Cash Flows Example

- Suppose you invest \$500 in a mutual fund today and \$600 in one year.
- If the fund pays 9% annually, how much will you have in two years?

$$\begin{aligned}
 \text{FV} &= \$ 500 \times (1.09)^2 = \$ 594.05 \\
 &+ \$ 600 \times (1.09) = \$ 654.00 \\
 &= \underline{\$1,248.05}
 \end{aligned}$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-10

Example Continued

- How much will you have in 5 years if you make no further deposits?
- First way:
 - $FV = \$500(1.09)^5 + \$600(1.09)^4 = \$1,616.26$
- Second way – use value at year 2:
 - $FV = \$1,248.05(1.09)^3 = \$1,616.26$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-11

Future Value: Multiple Cash Flows Example 3 - Formula

- Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years.
- How much will be in the account in five years if the interest rate is 8%?

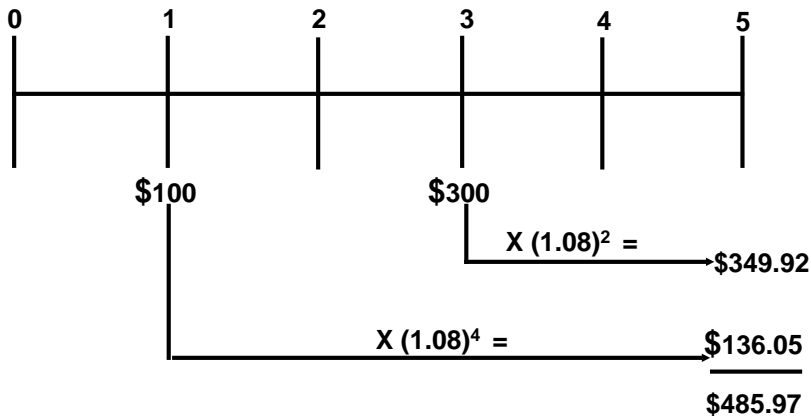
$$FV = \$100(1.08)^4 + \$300(1.08)^2 = \$136.05 + \$349.92 = \$485.97$$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-12

Example 3 Time Line



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-13

Present Value: Multiple Cash Flows Example 5.3

- You are offered an investment that will pay
- \$200 in year 1,
 - \$400 the next year,
 - \$600 the following year, and
 - \$800 at the end of the 4th year.
 - You can earn 12 percent on similar investments.
 - What is the most you should pay for this one?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-14

Present Value: Multiple Cash Flows

Example 5.3 - Formula

Find the PV of each cash flow and add them:

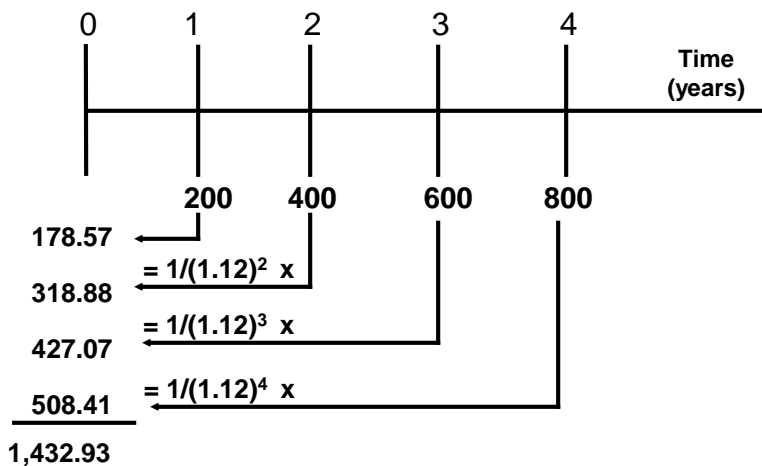
- Year 1 CF: $\$200 / (1.12)^1 = \$ 178.57$
- Year 2 CF: $\$400 / (1.12)^2 = \$ 318.88$
- Year 3 CF: $\$600 / (1.12)^3 = \$ 427.07$
- Year 4 CF: $\$800 / (1.12)^4 = \$ 508.41$
- Total PV $= \$1,432.93$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-15

Example 5.3 Time Line



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-16

Multiple Uneven Cash Flows

TI BAI + CF Worksheet

- Clear all:
 - Press **CF**
 - Then **2nd**
 - Then **CE/C**
- **CF₀** is displayed as **0.00**
- Enter the Period 0 cash flow
 - If an outflow, press **+/-** to change the sign
- To enter the figure in the cash flow register, press **ENTER**

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-17



TI BAI+: Uneven Cash Flows

- Press the down arrow to move to the next cash flow register
- Enter the cash flow amount, press **ENTER** and the down arrow to move to the cash flow counter (**Fnn**)
- The default counter value is “1”
 - To accept the value of “1”, press the down arrow again
 - To change the counter, enter the correct count, press **ENTER** and then the down arrow

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-18



TI BAI+ : Uneven Cash Flows

- Repeat for all cash flows, in order.
- To find NPV:
 - Press **NPV**: I appears on the screen.
 - Enter the interest rate, press **ENTER**, and then the down arrow to display NPV.
 - Press **CPT**.

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-19

TI BAI+ : Uneven Cash Flows

<u>Cash Flows:</u>	<u>Display</u>	<u>You Enter</u>
CF0 = 0	C00	CF 2 nd CE/C 0 ENTER
CF1 = 200	C01	200 ENTER
CF2 = 400	F01	1 ENTER
CF3 = 600	C02	400 ENTER
CF4 = 800	F02	1 ENTER
	C03	600 ENTER
	F03	1 ENTER
	C04	800 ENTER
	F04	1 ENTER NPV
	I	12 ENTER down
	NPV	CPT
	1432.93	

Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-20

Present Value: Multiple Cash Flows Another Example – Formula Solution

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years.
- If you want to earn 10% on your money, how much would you be willing to pay?
 - $PV = \$1,000 / (1.1)^1 = \$ 909.09$
 - $PV = \$2,000 / (1.1)^2 = \$1,652.89$
 - $PV = \$3,000 / (1.1)^3 = \$2,253.94$
 - $PV = \underline{\underline{\$4,815.92}}$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-21

Decisions, Decisions

- Your broker calls you and tells you that he has this great investment opportunity.
- If you invest \$100 today, you will receive \$40 in one year and \$75 in two years.
- If you require a 15% return on investments of this risk, should you take the investment?
- No – the broker is charging more than you would be willing to pay.

Use cash flow keys:

	CF
	2nd CE/C
CF0	0 ENTER
C01	40 ENTER
F01	1 ENTER
C02	75 ENTER
F02	1 ENTER
	NPV
I	15 ENTER
	DOWN CPT
	91.49

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-22

Saving For Retirement

- You are offered the opportunity to put some money away for retirement. You will receive five annual payments of \$25,000 each beginning in 40 years.

How much would you be willing to invest today if you desire an interest rate of 12%?

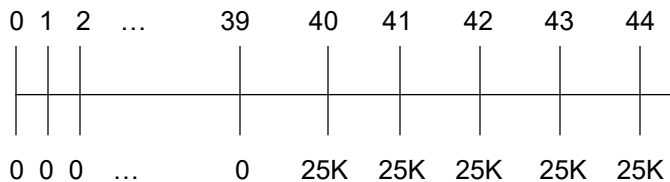
Use cash flow keys:

	CF
	2 nd CE/C
CF0	0 ENTER
C01	0 ENTER
F01	39 ENTER
C02	25000 ENTER
F02	5 ENTER
	NPV
I	12 ENTER
	DOWN
	CPT
	1084.71

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-23

Saving For Retirement Time Line



Notice that the year 0 cash flow = 0 ($CF_0 = 0$)

Cash flows years 1–39 = 0 ($C01 = 0$; $F01 = 39$)

Cash flows years 40–44 = 25,000 ($C02 = 25,000$; $F02 = 5$)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-24

Quick Quiz: Part 1

- Suppose you are looking at the following possible cash flows:
 - Year 1 CF = \$100;
 - Years 2 and 3 CFs = \$200;
 - Years 4 and 5 CFs = \$300.
 - The required discount rate is 7%
- What is the value of the CFs at year 5?
- What is the value of the CFs today?

Calculator Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-25

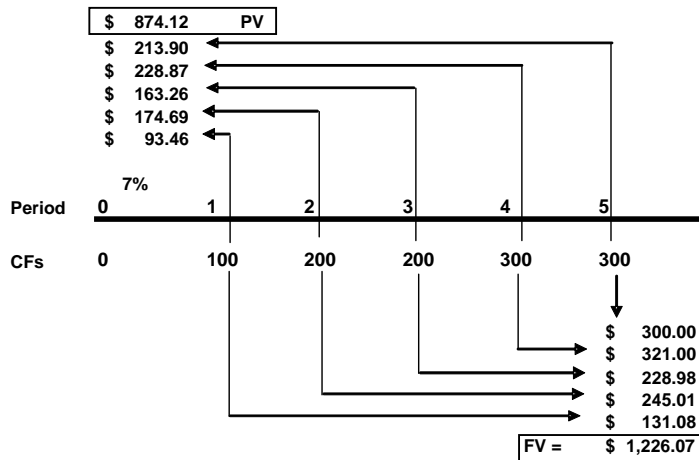
Quick Quiz: Part 1 – Excel Solution

	A	B	C	D	E
1	Chapter 5 - Quick Quiz 1				
2		Rate	7%		
3	Year	Nper	CF	PV	Formula
4	1	1	100	\$93.46	=-PV(\$C\$2,A4,0,C4)
5	2	2	200	\$174.69	=-PV(\$C\$2,A5,0,C5)
6	3	3	200	\$163.26	=-PV(\$C\$2,A6,0,C6)
7	4	4	300	\$228.87	=-PV(\$C\$2,A7,0,C7)
8	5	5	300	\$213.90	=-PV(\$C\$2,A8,0,C8)
9			Total PV	\$874.17	=SUM(C4:C8)
10					
11	Year	Nper	CF	FV	Year
12	1	4	100	\$131.08	=-FV(\$C\$2,B12,0,C12)
13	2	3	200	\$245.01	=-FV(\$C\$2,B13,0,C13)
14	3	2	200	\$228.98	=-FV(\$C\$2,B14,0,C14)
15	4	1	300	\$321.00	=-FV(\$C\$2,B15,0,C15)
16	5	0	300	\$300.00	=-FV(\$C\$2,B16,0,C16)
17			Total FV	\$1,226.07	=SUM(C12:C16)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-26

Chapter 5 – Quick Quiz: Part 1



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-27

Annuities and Perpetuities

- **Annuity** – finite series of equal payments that occur at regular intervals
 - If the first payment occurs at the end of the period, it is called an *ordinary annuity*
 - If the first payment occurs at the beginning of the period, it is called an *annuity due*
- **Perpetuity** – infinite series of equal payments.

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-28

Annuities and Perpetuities

Basic Formulas

- Perpetuity: $PV = PMT / r$

- Annuities:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = PMT \left[\frac{(1+r)^t - 1}{r} \right]$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-29

Annuities and the Calculator

- The **PMT** key on the calculator is used for the equal payment
- The sign convention still holds
- Ordinary annuity versus Annuity due
 - Switch your calculator between the two types (next slide)
 - If you see “**BGN**” or “**Begin**” in the display of your calculator, you have it set for an annuity due
 - Most problems are ordinary annuities

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-30

TI BAI+: Set Annuity Time Value Parameters

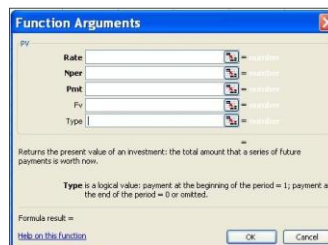
- Set END for an ordinary annuity or BGN for an annuity due
 - Press **2nd BGN** (above **PMT**)
 - This is a toggle switch. The default is END.
 - To change to BEGIN, press **2nd SET** (above **ENTER**) to go back and forth.

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-31

Excel Spreadsheet Functions

- FV(Rate,Nper,Pmt,PV,**0/1**)
- PV(Rate,Nper,Pmt,FV,**0/1**)
- RATE(Nper,Pmt,PV,FV,**0/1**)
- NPER(Rate,Pmt,PV,FV,**0/1**)
- PMT(Rate,Nper,PV,FV,**0/1**)



- Inside parens: (RATE,NPER,PMT,PV,FV,**0/1**)
- **“0/1”** Ordinary annuity = 0 (default; no entry needed)
Annuity Due = 1 (must be entered)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-32

Important Points to Remember

- Interest rate and time period must match!
 - Annual periods \Rightarrow annual rate
 - Monthly periods \Rightarrow monthly rate
- The Sign Convention
 - Cash inflows are positive
 - Cash outflows are negative

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-33

Sign Convention Example

5	N	5	N
10	I/Y	10	I/Y
-100	PV	-100	PV
20	PMT	-20	PMT
CPT FV = \$38.95		CPT FV = \$283.15	
Implies you deposited \$100 today and plan to WITHDRAW \$20 a year for 5 years		Implies you deposited \$100 today and plan to ADD \$20 a year for 5 years	
+CF = Cash INFLOW to YOU		-CF = Cash OUTFLOW from you	

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-34

Annuity Example 5.5

- You can afford \$632 per month.
- Going rate = 1%/month for 48 months.
- How much can you borrow?
- You borrow money TODAY so you need to compute the present value.

48	<i>N</i>
1	<i>I/Y</i>
632	<i>PMT</i>
0	<i>FV</i>
<i>CPT PV</i> = 23,999.54	
(\$24,000)	

$$PV = 632 \left[\frac{1 - \frac{1}{(1.01)^{48}}}{.01} \right] = 23,999.54$$

$$=PV(0.01,48,-632,0)$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-35

Annuity: Sweepstakes Example

- Suppose you win the Publishers Clearinghouse \$10 million sweepstakes.
- The money is paid in equal annual installments of \$333,333.33 over 30 years.
- If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?
 - $PV = \$333,333.33[1 - 1/1.05^{30}] / .05 = \$5,124,150.29$

Calculator and Excel Solution

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-36

Buying a House

- You are ready to buy a house and you have \$20,000 for a down payment and closing costs.
- Closing costs are estimated to be 4% of the loan value.
- You have an annual salary of \$36,000.
- The bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.
- The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.
- How much money will the bank loan you?
- How much can you offer for the house?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-37

Buying a House (continued)

- Bank loan
 - Monthly income = $36,000 / 12 = 3,000$
 - Maximum payment = $.28(3,000) = 840$
 - 360 N (30*12)
 - 0.5 I/Y
 - -840 PMT
 - **CPT PV = 140,105**
- Total Price
 - Closing costs = $.04(140,105) = 5,604$
 - Down payment = $20,000 - 5,604 = 14,396$
 - Total Price = $140,105 + 14,396 = \mathbf{154,501}$

=PV(.005,360,-840,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-38

Quick Quiz: Part 2

- You know the payment amount for a loan and you want to know how much was borrowed.
 - Do you compute a present value or a future value?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-39

Quick Quiz: Part 2

- You want to receive \$5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?
 - 300 N ← Months
 - 0.75 I/Y ← Monthly rate
 - 5000 PMT ← Monthly Payment
 - 0 FV
 - **CPT PV = -595,808.11**

=PV(0.0075,300,5000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-40

Finding the Payment

- Suppose you want to borrow \$20,000 for a new car.
- You can borrow at 8% per year, compounded monthly ($8/12 = .66667\%$ per month).
- If you take a 4 year loan, what is your monthly payment?

4(12) = 48	N
0.66667	I/Y
20,000	PV
0	FV
CPT PMT = -488.26	

=PMT(0.006667,48,20000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-41

Finding the Number of Payments

Example 5.6

- \$1,000 due on credit card
- Payment = \$20 month minimum
- Rate = 1.5% per month
- **The sign convention matters!!!**

1.5	I/Y
1000	PV
-20	PMT
0	FV
CPT N = 93.111 months	
= 7.75 years	

=NPER(0.015,-20,1000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-42

Finding the Number of Payments

Another Example

- Suppose you borrow \$2,000 at 5% and you are going to make annual payments of \$734.42. How long before you pay off the loan?

5	I/Y
2000	PV
-734.42	PMT
0	FV
CPT N = 3 years	

=NPER(0.05,-734.42,2000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-43

Finding the Rate

- Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?

60	N
10000	PV
-207.58	PMT
0	FV
CPT I/Y = 0.75%	
per month	

=RATE(60,-207.58,10000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-44

Quick Quiz: Part 3

- You want to receive \$5,000 per month for the next 5 years. How much would you need to deposit today if you can earn .75% per month?

60	N (months)
0.75	I/Y
5000	PMT
0	FV
CPT PV = -240866.87	

=PV(0.0075,60,5000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-45

Quick Quiz: Part 3

- You want to receive \$5,000 per month for the next 5 years.
- What monthly rate would you need to earn if you only have \$200,000 to deposit?

60	N
-200000	PV
5000	PMT
0	FV
CPT I/Y = 1.4395%	
per month	

=RATE(60,5000,-200000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-46

Quick Quiz: Part 3

- Suppose you have \$200,000 to deposit and can earn .75% per month.
 - How many months could you receive the \$5,000 payment?

0.75	I/Y
-200000	PV
5000	PMT
0	FV
CPT N = 47.73 months	
≈ 4 years	

=NPER(0.0075,5000,-200000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-47

Quick Quiz: Part 3

- Suppose you have \$200,000 to deposit and can earn .75% per month.
 - How much could you receive every month for 5 years?

60	N
0.75	I/Y
-200000	PV
0	FV
CPT PMT = 4151.67	

=PMT(0.0075,60,-200000,0)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-48

Future Values for Annuities

- Suppose you begin saving for your retirement by depositing \$2,000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?

40	N
7.5	I/Y
0	PV
-2000	PMT
CPT FV = 454513.04	

=FV(0.075,40,-2000,0)

$$FV = PMT \left[\frac{(1+r)^t - 1}{r} \right]$$

$$FV = 2000 \left[\frac{(1.075)^{40} - 1}{.075} \right] = 454,513.04$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-49

Annuity Due

- You are saving for a new house and you put \$10,000 per year in an account paying 8%. The first payment is made **today**. How much will you have at the end of 3 years?

2nd BGN 2nd SET

3	N
8	I/Y
0	PV
-10000	PMT
CPT FV = 35061.12	
2nd BGN 2nd SET	

=FV(0.08,3,-10000,0,1)

$$FV_{AD} = PMT \left[\frac{(1+r)^t - 1}{r} \right] (1+r)$$

$$FV_{AD} = 10000 \left[\frac{(1.08)^3 - 1}{.08} \right] (1.08) = 35,061.12$$

◀ Reset to END

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-50

Table 5.2

I. Symbols

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future at Time *t*

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of C Invested per period for *t* periods at *r* percent per period

$$FV_t = C \times [(1 + r)^t - 1]/r$$

A series of identical cash flows paid for a set number of periods is called an annuity, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present value of C per period for *t* periods at *r* percent per period

$$PV = C \times [1 - 1/(1 + r)^t]/r$$

The term $[1 - 1/(1 + r)^t]/r$ is called the *annuity present value factor*.

IV. Present value of a perpetuity of C per period

$$PV = C/r$$

A perpetuity has the same cash flow every period forever.

TABLE 5.2

Summary of annuity and perpetuity calculations

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-51

Example: Work the Web

- Another online financial calculator can be found at [Calculatoredge.com](https://www.calculatoredge.com).
- Click [on this link](#), select “Finance” calculator and “Annuity Payments” and work the following example:
 - How much could you withdraw each year if you have \$2,500,000, earn 8% and make annual withdrawals for 35 years?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-52

Perpetuity Example 5.7

- Perpetuity formula: $PV = PMT / r$
- Current required return:
 - $40 = 1 / r$
 - $r = .025$ or 2.5% per quarter
- Dividend for new preferred:
 - $100 = PMT / .025$
 - $PMT = 2.50$ per quarter

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-53

Quick Quiz: Part 4 (1 of 3)

- You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if **the first payment is made in one month?**

420	N
1	I/Y
0	PV
1000000	FV
CPT PMT = -155.50	

Ordinary Annuity

=PMT(0.01,420,0,1000000)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-54

Quick Quiz: Part 4 (2 of 3)

- You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if **the first payment is made today?**

2nd BGN 2nd SET
420 N
1 I/Y
0 PV
1000000 FV
CPT PMT = -153.96
2nd BGN 2nd SET

Annuity Due

=PMT(0.01,420,0,1000000,1)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-55

Quick Quiz: Part 4 (3 of 3)

- You are considering preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

$$\$1.50/0.03 = \$50$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-56

Interest Rates

- **Effective Annual Rate (EAR)**
 - The interest rate expressed as if it were compounded once per year.
 - Used to compare two alternative investments with different compounding periods
- **Annual Percentage Rate (APR) “Nominal”**
 - The annual rate quoted by law
 - $\text{APR} = \text{periodic rate} \times \text{number of periods per year}$
 - $\text{Periodic rate} = \text{APR} / \text{periods per year}$

← Return to Quick Quiz

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-57

Things to Remember

- You ALWAYS need to make sure that the interest rate and the time period match.
 - Annual periods → annual rate.
 - Monthly periods → monthly rate.
- If you have an APR based on monthly compounding, you have to use monthly periods for lump sums or adjust the interest rate accordingly.

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-58

EAR Formula

$$\text{EAR} = \left[1 + \frac{\text{APR}}{m} \right]^m - 1$$

APR = the quoted rate

m = number of compounds per year

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-59

EAR and APR in TI BA II+

2nd ICONV

2nd CE/C (to clear the memory)

- 3 fields in worksheet:
 - NOM (Nominal rate-APR)
 - EFF (Effective annual rate)
 - C/Y (Compounding periods/yr)

- To compute EFF, enter the NOM and C/Y values, move to EFF and press **CPT**
- To compute NOM, enter the EFF and C/Y values, move to NOM and press **CPT**

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-60

EAR and NOM in Excel

- 2 Functions:

=EFFECT(Nom, Nper)

=NOMINAL(Eff, Nper)

- All rates entered as decimals
- Nper = number of compounding periods per year

TOOLS ... Add-Ins ... ANALYSIS TOOLPAK

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-61

Decisions, Decisions

- Which savings accounts should you choose:
 - 5.25% with daily compounding.
 - 5.30% with semiannual compounding.
- First account:
 - $EAR = (1 + .0525/365)^{365} - 1 = 5.39\%$
 - ICONV: NOM=5.25; C/Y=365 **EFF=5.3899**
 - Excel: =EFFECT(0.525,365) = 5.39%
- Second account:
 - $EAR = (1 + .053/2)^2 - 1 = 5.37\%$
 - ICONV: NOM=5.3; C/Y=2 **EFF=5.3702**
 - Excel: =EFFECT(0.53,2) = 5.37%

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-62

Computing APRs

- What is the APR if the monthly rate is .5%?
 - $.5\%(12) = 6\%$
- What is the APR if the semiannual rate is .5%?
 - $.5\%(2) = 1\%$
- What is the monthly rate if the APR is 12% with monthly compounding?
 - $12\% / 12 = 1\%$
 - Can you divide the above APR by 2 to get the semiannual rate?
 - NO. You need an APR based on semiannual compounding to find the semiannual rate.

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-63

Computing EAR and APR

- Suppose you can earn 1% per month on \$1 invested today.
 - What is the APR? $1(12) = 12\%$
 - How much are you effectively earning?
 - $FV = 1(1.01)^{12} = 1.1268$
 - $\text{Rate} = (1.1268 - 1) / 1 = .1268 = \mathbf{12.68\%}$

INCONV:	NOM	= 12
	C/Y	= 12
	EFF	= 12.6825

=EFFECT(0.12,12)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-64

Computing EAR and APR

- Suppose if you put it in another account, you earn 3% per quarter.
 - What is the APR? $3(4) = 12\%$
 - How much are you effectively earning?
 - $FV = 1(1.03)^4 = 1.1255$
 - $\text{Rate} = (1.1255 - 1) / 1 = .1255 = 12.55\%$

ICONV:	NOM = 12
	C/Y = 4
	EFF = 12.5509
=EFFECT(0.12,4)	

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-65

Computing APRs from EARs

$$APR = m \left[(1 + EAR)^{1/m} - 1 \right]$$

M = number of compounding periods per year

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-66

APR - Example

- Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$\text{APR} = 12 \left[(1 + .12)^{1/12} - 1 \right] = .1138655 \text{ or } 11.39\%$$

$$\text{ICONV: EFF} = 12$$

$$\text{C/Y} = 12$$

$$\text{NOM} = 11.3866$$

$$\text{Excel: =NOMINAL(0.12,12)}$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-67

Computing Payments with APRs

- Suppose you want to buy a new computer.
- The store is willing to allow you to make monthly payments.
- The entire computer system costs \$3,500.
- The loan period is for 2 years.
- The interest rate is 16.9% with monthly compounding.
- What is your monthly payment?

2(12)	= 24	N
16.9 / 12 = 1.40833		I/Y
3500		PV
0		FV
CPT PMT = -172.88		

$$\text{=PMT(0.0140833,24,3500,0)}$$

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-68

Future Values with Monthly Compounding

- Suppose you deposit \$50 a month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?

420	N	(35*12)	=FV(0.0075,420,-50,0)
0.75	I/Y	(9/12)	
0	PV		
-50	PMT		
CPT FV = 147,089.22			

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-69

Present Value with Daily Compounding

- You need \$15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?

1095	N	(3*365)
0.15068493	I/Y	(5.5/365)
0	PMT	
15,000	FV	
CPT PV = -12,718.56		

=PV(0.00015,1095,0,15000)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-70

Quick Quiz: Part 5

- What is the definition of an APR?
- What is the effective annual rate?
- Which rate should you use to compare alternative investments or loans?
- Which rate do you need to use in the time value of money calculations?

(Answers = Slide 5.56)

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-71

Pure Discount Loans

- Treasury bills are excellent examples of pure discount loans.
 - Principal amount is repaid at some future date
 - No periodic interest payments
- If a T-bill promises to repay \$10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?
 - **1 N; 10,000 FV; 7 I/Y; CPT PV = -9345.79**
 - **=PV(.07,1,0,10000)**



Return to
Quick Quiz

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-72

Amortized Loan with Fixed Payment: Example

- Each payment covers the interest expense plus reduces principal
- Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5000.

– What is the annual payment?

← Return to Quick Quiz

- $5,000 = \text{PMT}[1 - 1 / 1.08^4] / .08 \rightarrow \text{PMT} = 1,509.60$
- $= \text{PMT}(0.08, 4, 5000, 0) = 1509.60$
- 4 N; 8 I/Y; 5000 PV, 0 FV, CPT PMT = 1509.60

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-73

Amortized Loan with Fixed Payment: Example

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$ 5,000.00	\$ 1,509.60	\$ 400.00	\$ 1,109.60	\$ 3,890.40
2	\$ 3,890.40	\$ 1,509.60	\$ 311.23	\$ 1,198.37	\$ 2,692.03
3	\$ 2,692.03	\$ 1,509.60	\$ 215.36	\$ 1,294.24	\$ 1,397.79
4	\$ 1,397.79	\$ 1,509.60	\$ 111.82	\$ 1,397.79	\$ -
Totals		\$ 6,038.40	\$ 1,038.42	\$ 5,000.00	

Interest Paid = Beginning Balance * Rate (8%)

Principal Paid = Total Payment – Interest Paid

Ending Balance = Beginning Balance – Principal Paid



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-74

Quick Quiz: Part 6

- What is a pure discount loan?
 - What is a good example of a pure discount loan? ([Slide 5.72](#))
- What is an amortized loan?
 - What is a good example of an amortized loan? ([Slide 5.73](#))

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-75

Example: Work the Web

- Several Web sites have calculators that will prepare amortization tables quickly
- One such site is Bankrate.com
- Click [on this link](#), select “Calculators,” “Mortgage Payment Calculator,” and enter the following information:
 - Loan amount = \$20,000
 - Term = 10 years
 - Interest rate = 7.625%
 - What is the monthly payment?

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-76

FV Example 5.1 Calculator Solution

Calculator Solution						
Year	N	I/Y	PV	PMT	CPT FV	
0	3	8	-7000	0	8,817.98	
1	2	8	-4000	0	4,665.60	
2	1	8	-4000	0	4,320.00	
3					4,000.00	
					21,803.58	
Value at year 4:						
Year	N	I/Y	PV	PMT	CPT FV	
4	1	8	-21,803.58	0	23,547.87	

Return to
Slideshow

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-77

FV Example 5.1 Excel Solution

Excel Solution						
Year	Nper	Rate	PV	PMT	FV	
0	3	0.08	-7000	0	8,817.98	
1	2	0.08	-4000	0	4,665.60	
2	1	0.08	-4000	0	4,320.00	
3					4,000.00	
					21,803.58	
Value at year 4:						
Year	Nper	Rate	PV	PMT	FV	
4	1	0.08	-21,803.58	0	23,547.87	

=FV(Rate, Nper, PMT, PV)

Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-78

FV Example 2 Calculator Solution

Calculator Solution						
	Year	N	I/Y	PV	PMT	CPT FV
	0	2	9	500	0	594.05
	1	1	9	600	0	654.00
						1,248.05
Value at year 4:						
	Year	N	I/Y	PV	PMT	CPT FV
	5	3	9	1,248.05	0	1,616.26
or						
	Year	N	I/Y	PV	PMT	CPT FV
	0	5	9	500	0	769.31
	1	4	9	600	0	846.95
						1,616.26


Return to
Slideshow

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-79

FV Example 2 Excel Solution

Excel Solution						
	Year	Nper	Rate	PV	PMT	FV
	0	2	0.09	-500	0	594.05
	1	1	0.09	-600	0	654.00
						1,248.05
Value at year 4:						
	Year	Nper	Rate	PV	PMT	FV
	5	3	0.09	-1,248.05	0	1,616.26
=FV(Rate, Nper, PMT, PV)						


Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-80

FV Example 3 Calculator and Excel Solution

Calculator Solution						CPT
Year	N	I/Y	PV	PMT		FV
1	4	8	-100	0		136.05
3	2	8	-300	0		349.92
						485.97
Excel Solution						FV
Year	Nper	Rate	PV	PMT		FV
1	4	0.08	-100	0		136.05
3	2	0.08	-300	0		349.92
						485.97
=FV(Rate, Nper,PMT,PV)						



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-81



Multiple Cash Flows: Example 5.3 Calculator Solution

Year	N	I/Y	FV	PMT	CPT
					PV
1	1	12	200	0	178.57
2	2	12	400	0	318.88
3	3	12	600	0	427.07
4	4	12	800	0	508.41
					1,432.93



Return to
Slideshow

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-82



Multiple Cash Flows: Example 5.3 Excel Solution

Excel Solution

Year	Nper	Rate	FV	PMT	PV
1	1	0.12	-200	0	178.57
2	2	0.12	-400	0	318.88
3	3	0.12	-600	0	427.07
4	4	0.12	-800	0	508.41
					1,432.93

=PV(Rate, Nper,PMT,FV)



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-83

Excel: PV of Multiple Uneven CFs

Rate	12%		Present Value	Formula
Period	Cash Flow			
1	\$ 200.00		(\$178.57)	=PV(\$B\$1,A3,0,B3)
2	\$ 400.00		(\$318.88)	=PV(\$B\$1,A4,0,B4)
3	\$ 600.00		(\$427.07)	=PV(\$B\$1,A5,0,B5)
4	\$ 800.00		(\$508.41)	=PV(\$B\$1,A6,0,B6)
Total PV =			(\$1,432.93)	=SUM(C3:C6)
			(\$1,432.93)	=NPV(B1,B3:B6)

The functions require a PMT = 0.



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-84

Multiple Cash Flows: PV Example Calculator & Excel Solutions

Calculator Solution						CPT
Year	N	I/Y	FV	PMT	PV	
1	1	10	-1000	0	909.09	
2	2	10	-2000	0	1,652.89	
3	3	10	-3000	0	2,253.94	
						4,815.92

Excel Solution						
Year	Nper	Rate	FV	PMT	PV	
1	1	0.10	-1000	0	909.09	
2	2	0.10	-2000	0	1,652.89	
3	3	0.10	-3000	0	2,253.94	
						4,815.92

=PV(Rate, Nper, PMT, FV)



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-85

Quick Quiz: Part 1

Discount Rate	7%	Calculator:						
Year	CF	Keystrokes						
1	100	CF	2nd	ClrWork				
2	200	CF0	0	ENTER				
3	200	C01	100	ENTER	F01	1	ENTER	
4	300	C02	200	ENTER	F02	2	ENTER	
5	300	C03	300	ENTER	F03	2	ENTER	
		NPV		ENTER				
		I	7	ENTER				
		DOWN	CPT					874.17
			Year 3	Year 5				
		N	3	5				
		I/Y	7	7				
		PV	-874.17	-874.17				
		PMT	0	0				
		CPT FV	1070.89	1226.07				



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-86

Annuity – Sweepstakes Example

Sweepstakes Example

N	30
I/Y	5
PMT	\$ 333,333.33
FV	0
CPT PV	\$ (5,124,150.29)

$$=PV(5, 30, 333333.33, 0) = (\$5,124,150.29)$$



Return to
Slideshow



Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-87

Chapter 5

END

Copyright ©2020 McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

5-88