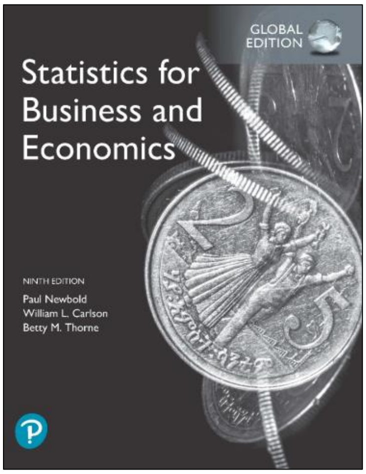
Statistics for Business and Economics

Ninth Edition, Global Edition



Chapter 10

Hypothesis Testing: Additional Topics

1

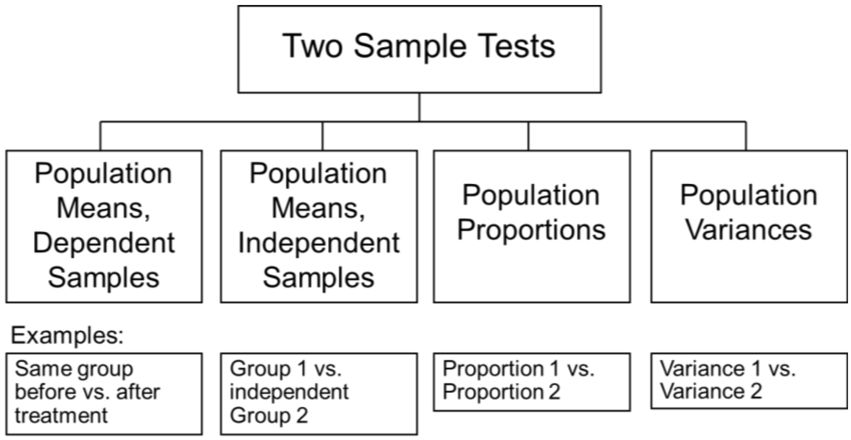
Chapter Goals

After completing this chapter, you should be able to:

- Test hypotheses for the difference between two population means
  - Two means, matched pairs
  - Independent populations, population variances known
  - Independent populations, population variances unknown but equal
- Complete a hypothesis test for the difference between two proportions (large samples)
- Use the *F* table to find critical *F* values
- Complete an *F* test for the equality of two variances

2

Two Sample Tests



 Slide - 3

3

Section 10.1 Dependent Samples

Dependent Samples

Tests of the Difference Between Two Normal Population Means: Dependent Samples

Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$d_i = x_i - y_i$$

- Assumptions:
  - Both Populations Are Normally Distributed

 Slide - 4

4

Test Statistic: Dependent Samples

Population Means, Dependent Samples

For tests of the following form:

$$H_0: \mu_x - \mu_y \geq 0$$
$$H_0: \mu_x - \mu_y \leq 0$$
$$H_0: \mu_x - \mu_y = 0$$

The test statistic for the mean difference is a *t* value, with *n* − 1 degrees of freedom:

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} \qquad \text{where } \bar{d} = \frac{\sum d_i}{n}$$

s_d = sample standard dev. of differences

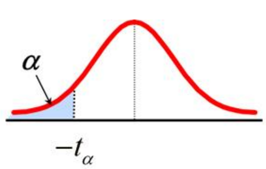
n = the sample size (number of pairs)

Decision Rules: Matched Pairs

Matched or Paired Samples

Lower-tail test:

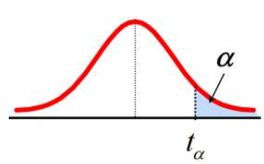
$$H_0: \mu_x - \mu_y \geq 0$$
$$H_1: \mu_x - \mu_y < 0$$



Reject H_0 if $t < -t_{n-1,\alpha}$

Upper-tail test:

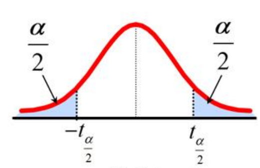
$$H_0: \mu_x - \mu_y \leq 0$$
$$H_1: \mu_x - \mu_y > 0$$



Reject H_0 if $t > t_{n-1,\alpha}$

Two-tail test:

$$H_0: \mu_x - \mu_y = 0$$
$$H_1: \mu_x - \mu_y \neq 0$$



Reject H_0 if $t < -t_{n-1,\frac{\alpha}{2}}$ or $t > t_{n-1,\frac{\alpha}{2}}$

Where $t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$ has $n-1$ d.f.

Matched Pairs Example

- Assume you send your salespeople to a “customer service” training workshop. Has the training made a difference in the number of complaints? You collect the following data:

Salesperson	Number of Complaints: Before (1)	After (2)	(2) − (1) Difference, d_i
C.B.	6	4	− 2
T.F.	20	6	− 14
M.H.	3	2	− 1
R.K.	0	0	0
M.O.	4	0	− 4
			−21

$$\bar{d} = \frac{\sum d_i}{n} = -4.2$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}} = 5.67$$

 Slide - 7

Matched Pairs: Solution

- Has the training made a difference in the number of complaints (at the $\alpha = 0.05$ level)?

$H_0 : \mu_x - \mu_y = 0$

$H_1 : \mu_x - \mu_y \neq 0$

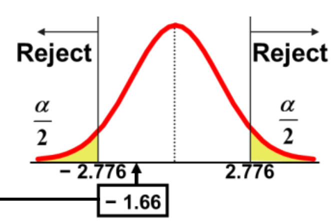
$\alpha = .05 \quad \bar{d} = -4.2$

Critical Value $= \pm 2.776$

$\text{d.f.} = n - 1 = 4$

Test Statistic:

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = \frac{-4.2}{\frac{5.67}{\sqrt{5}}} = -1.66$$



Decision: Do not reject H_0

(t stat is not in the reject region)

Conclusion: There is not a significant change in the number of complaints.

 Slide - 8

Section 10.2 Independent Samples

Population means, independent samples

Tests of the Difference Between Two Normal Population Means: Dependent Samples

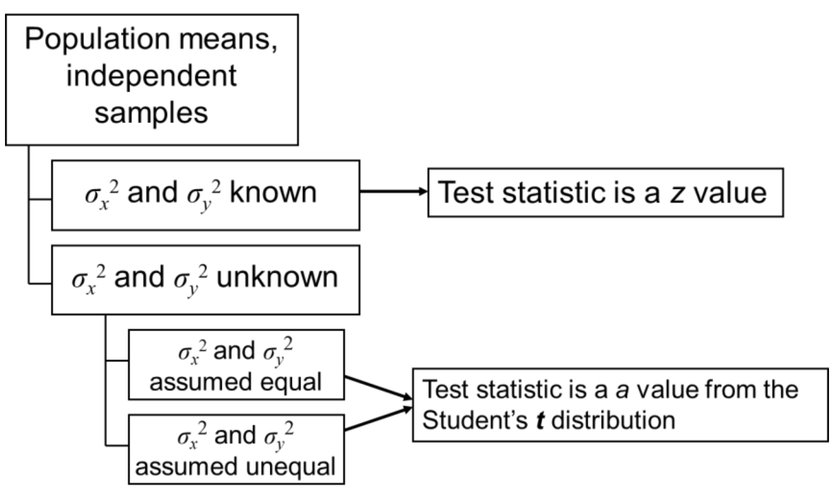
Goal: Form a confidence interval for the difference between two population means, $\mu_x - \mu_y$

- Different populations
  - Unrelated
  - Independent
    - Sample selected from one population has no effect on the sample selected from the other population
  - Normally distributed

 Slide - 9

9

Difference Between Two Means



 Slide - 10

10

Sigma Sub *x* Squared and Sigma Sub *y* Squared Known (1 of 2)

Population means, independent samples

- σ_x^2 and σ_y^2 known *
- σ_x^2 and σ_y^2 unknown

Assumptions:

- Samples are randomly and independently drawn
- both population distributions are normal
- Population variances are known

11

Sigma Sub *x* Squared and Sigma Sub *y* Squared Known (2 of 2)

Population means, independent samples

- σ_x^2 and σ_y^2 known *
- σ_x^2 and σ_y^2 unknown

When σ_x^2 and σ_y^2 are known and both populations are normal, the variance of $\bar{X} - \bar{Y}$ is

$$\sigma^2_{\bar{X}-\bar{Y}} = \frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}$$

…and the random variable

$$Z = \frac{(\bar{x} - \bar{y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma^2_x}{n_X} + \frac{\sigma^2_y}{n_Y}}}$$

has a standard normal distribution

12

Test Statistic, Sigma Sub *x* Squared and Sigma Sub *y* Squared Known

Population means, independent samples

- σ_x^2 and σ_y^2 known *
- σ_x^2 and σ_y^2 unknown

$$H_0 : \mu_x - \mu_y = 0$$

The test statistic for $\mu_x - \mu_y$ is:

$$z = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

Hypothesis Tests for Two Population Means

Two Population Means, Independent Samples

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0 : \mu_x \geq \mu_y$	$H_0 : \mu_x \leq \mu_y$	$H_0 : \mu_x = \mu_y$
$H_1 : \mu_x < \mu_y$	$H_1 : \mu_x > \mu_y$	$H_1 : \mu_x \neq \mu_y$
i.e.,	i.e.,	i.e.,
$H_0 : \mu_x - \mu_y \geq 0$	$H_0 : \mu_x - \mu_y \leq 0$	$H_0 : \mu_x - \mu_y = 0$
$H_1 : \mu_x - \mu_y < 0$	$H_1 : \mu_x - \mu_y > 0$	$H_1 : \mu_x - \mu_y \neq 0$

Decision Rules (1 of 2)

Two Population Means, Independent Samples, Variances Known

Lower-tail test:

$H_0: \mu_x - \mu_y \geq 0$

$H_1: \mu_x - \mu_y < 0$

Upper-tail test:

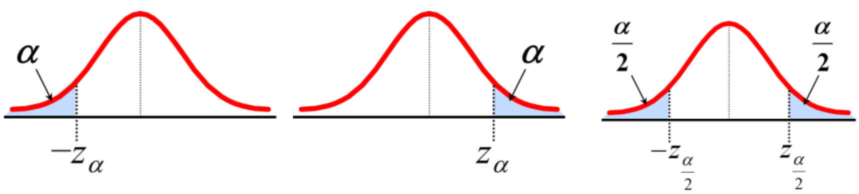
$H_0: \mu_x - \mu_y \leq 0$

$H_1: \mu_x - \mu_y > 0$

Two-tail test:

$H_0: \mu_x - \mu_y = 0$

$H_1: \mu_x - \mu_y \neq 0$



Reject H_0 if $z < -z_\alpha$

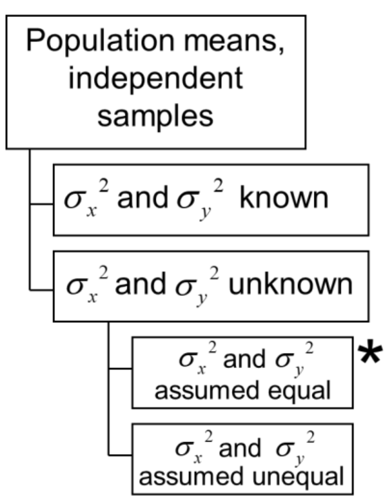
Reject H_0 if $z > z_\alpha$

Reject H_0 if $z < -z_{\frac{\alpha}{2}}$ or $z > z_{\frac{\alpha}{2}}$

 Slide - 15

15

Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Equal (1 of 2)



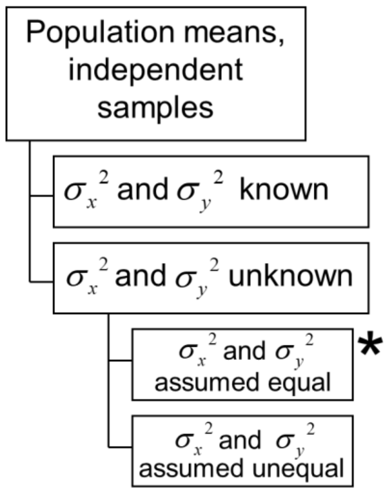
Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown but assumed equal

 Slide - 16

16

Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Equal (2 of 2)

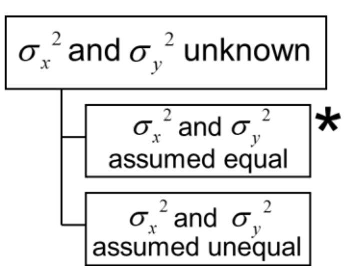


- The population variances are assumed equal, so use the two sample standard deviations and pool them to estimate σ
- use a *t* value with $(n_x + n_y - 2)$ degrees of freedom

 Slide - 17

17

Test Statistic, Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Equal



The test statistic for $H_0 : \mu_x - \mu_y = 0$ is:

$$t = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}}$$

Where *t* has $(n_1 + n_2 - 2)$ d.f., and

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

 Slide - 18

18

Decision Rules (2 of 2)

Two Population Means, Independent Samples, Variances Unknown

Lower-tail test:

$H_0: \mu_x - \mu_y \geq 0$

$H_1: \mu_x - \mu_y < 0$

Upper-tail test:

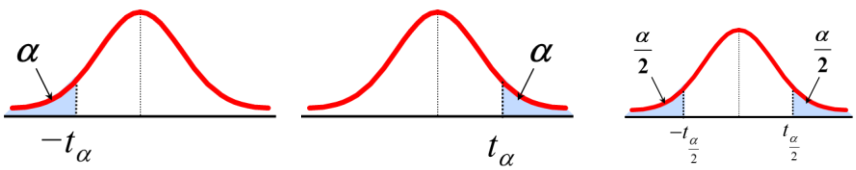
$H_0: \mu_x - \mu_y \leq 0$

$H_1: \mu_x - \mu_y > 0$

Two-tail test:

$H_0: \mu_x - \mu_y = 0$

$H_1: \mu_x - \mu_y \neq 0$



Reject H_0 if $t < -t_{(n1+n2-2),\ \alpha}$

Reject H_0 if $t > t_{(n1+n2-2),\ \alpha}$

Reject H_0 if $t < -t_{(n1+n2-2),\ \frac{\alpha}{2}}$ or $t > t_{(n1+n2-2),\ \frac{\alpha}{2}}$

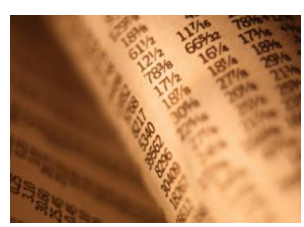
 Slide - 19

19

Pooled Variance *t* Test: Example

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	21	25
Sample mean	3.27	2.53
Sample std dev	1.30	1.16



Assuming both populations are approximately normal with equal variances, is there a difference in average yield $(\alpha = 0.05)$?

 Slide - 20

20

Calculating the Test Statistic

$$H_0: \mu_1 - \mu_2 = 0 \text{ i.e. } (\mu_1 = \mu_2)$$
$$H_1: \mu_1 - \mu_2 \neq 0 \text{ i.e. } (\mu_1 \neq \mu_2)$$

The test statistic is:

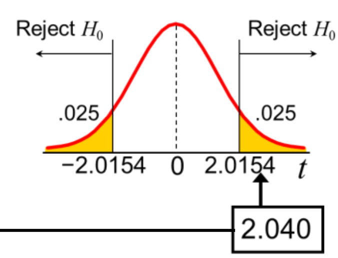
$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(3.27 - 2.53)}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(21-1)1.30^2 + (25-1)1.16^2}{(21-1) + (25-1)} = 1.5021$$

Solution

$$H_0: \mu_1 - \mu_2 = 0 \text{ i.e. } (\mu_1 = \mu_2)$$
$$H_1: \mu_1 - \mu_2 \neq 0 \text{ i.e. } (\mu_1 \neq \mu_2)$$
$$\alpha = 0.05$$
$$\text{df} = 21 + 25 - 2 = 44$$

Critical Values: $t = \pm 2.0154$



Test Statistic:

$$t = \frac{3.27 - 2.53}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = 2.040$$

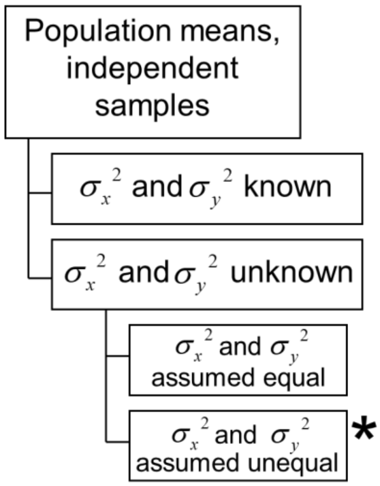
Decision:

Reject H_0 at $\alpha = 0.05$

Conclusion:

There is evidence of a difference in means.

Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Unequal (1 of 2)



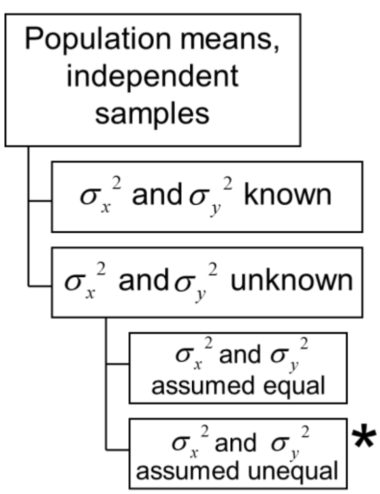
Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown and assumed unequal

 Slide - 23

23

Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Unequal (2 of 2)



Forming interval estimates:

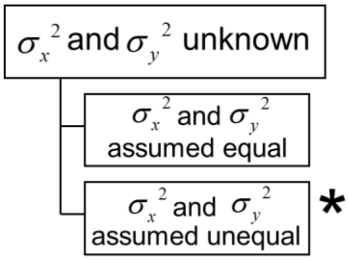
- The population variances are assumed unequal, so a pooled variance is not appropriate
- use a *t* value with *v* degrees of freedom, where

$$v = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x - 1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{n_y - 1}}$$

 Slide - 24

24

Test Statistic, Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Unequal



The test statistic for $H_0 : \mu_x - \mu_y = 0$ is:

$$t = \frac{(\bar{x} - \bar{y})}{\sqrt{\frac{s_x^2}{n_X} + \frac{s_y^2}{n_Y}}}$$

Where t has v degrees of freedom: $v = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x - 1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{n_y - 1}}$

 Slide - 25

25

Section 10.3 Two Population Proportions

Population proportions

Tests of the Difference Between Two Population Proportions (Large Samples)

Goal: Test hypotheses for the difference between two population proportions, $P_x - P_y$

Assumptions:

Both sample sizes are large,

$nP(1 - P) > 5$

 Slide - 26

26

Two Population Proportions

Population proportions

- The random variable

$$Z = \frac{(\hat{p}_x - \hat{p}_y) - (P_x - P_y)}{\sqrt{\frac{P_x(1-P_x)}{n_x} + \frac{P_y(1-P_y)}{n_y}}}$$

has a standard normal distribution

Pearson Slide - 27

27

Test Statistic for Two Population Proportions

Population proportions

The test statistic for

$H_0 : P_x - P_y = 0$

is a *z* value:

$$z = \frac{(\hat{p}_x - \hat{p}_y)}{\sqrt{\frac{\hat{p}_0(1-\hat{p}_0)}{n_x} + \frac{\hat{p}_0(1-\hat{p}_0)}{n_y}}}$$

Where $\hat{p}_0 = \frac{n_x \hat{p}_x + n_y \hat{p}_y}{n_x + n_y}$

Pearson Slide - 28

28

Decision Rules: Proportions

Population proportions

Lower-tail test:

$H_0: P_x - P_y \geq 0$

$H_1: P_x - P_y < 0$

Upper-tail test:

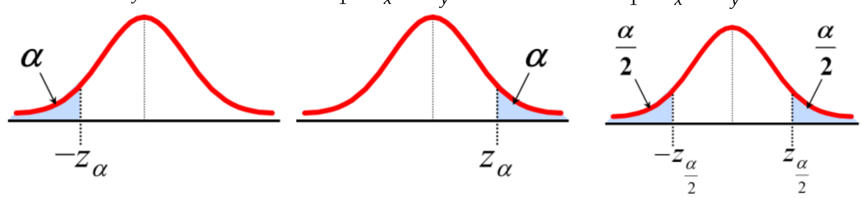
$H_0: P_x - P_y \leq 0$

$H_1: P_x - P_y > 0$

Two-tail test:

$H_0: P_x - P_y = 0$

$H_1: P_x - P_y \neq 0$



Reject H_0 if $z < -z_\alpha$

Reject H_0 if $z > z_\alpha$

Reject H_0 if $z < -z_{\frac{\alpha}{2}}$ or $z > z_{\frac{\alpha}{2}}$

 Slide - 29

29

Example 1: Two Population Proportions (1 of 3)

Is there a significant difference between the proportion of men and the proportion of women who will vote Yes on Proposition *A*?

- In a random sample, 36 of 72 men and 31 of 50 women indicated they would vote Yes
- Test at the .05 level of significance

 Slide - 30

30

Example 1: Two Population Proportions (2 of 3)

- The hypothesis test is:

$H_0: P_M - P_W = 0$ (the two proportions are equal)

$H_1: P_M - P_W \neq 0$ (there is a significant difference between proportions)

- The sample proportions are:
  - Men: $\hat{p}_M = \frac{36}{72} = .50$
  - Women: $\hat{p}_W = \frac{31}{50} = .62$
- The estimate for the common overall proportion is:

$$\hat{p}_0 = \frac{n_M \hat{p}_M + n_W \hat{p}_W}{n_M + n_W} = \frac{72\left(\frac{36}{72}\right) + 50\left(\frac{31}{50}\right)}{72 + 50} = \frac{67}{122} = .549$$

 Slide - 31

31

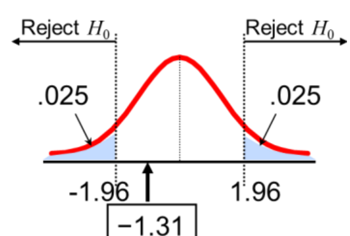
Example 1: Two Population Proportions (3 of 3)

The test statistic for $P_M - P_W = 0$ is:

$$z = \frac{(\hat{p}_M - \hat{p}_W)}{\sqrt{\frac{\hat{p}_0(1-\hat{p}_0)}{n_1} + \frac{\hat{p}_0(1-\hat{p}_0)}{n_2}}}$$

$$= \frac{(.50 - .62)}{\sqrt{\left(\frac{.549(1-.549)}{72} + \frac{.549(1-.549)}{50}\right)}}$$

$$= \boxed{-1.31}$$



Critical Values = ±1.96

For $\alpha = .05$

Decision: Do not reject H_0

Conclusion: There is not significant evidence of a difference between men and women in proportions who will vote yes.

 Slide - 32

32

Section 10.4 Tests of Equality of Two Variances

Tests for Two Population Variances

F test statistic

- Goal: Test hypotheses about two population variances

$H_0: \sigma_x^2 \geq \sigma_y^2$
$H_1: \sigma_x^2 < \sigma_y^2$ Lower-tail test

$H_0: \sigma_x^2 \leq \sigma_y^2$
$H_1: \sigma_x^2 > \sigma_y^2$ Upper-tail test

$H_0: \sigma_x^2 = \sigma_y^2$
$H_1: \sigma_x^2 \neq \sigma_y^2$ Two-tail test

The two populations are assumed to be independent and normally distributed

 Slide - 33

33

Hypothesis Tests for Two Variances

Tests for Two Population Variances

F test statistic

The random variable

$$F = \frac{\dfrac{s_x^2}{\sigma_x^2}}{\dfrac{s_y^2}{\sigma_y^2}}$$

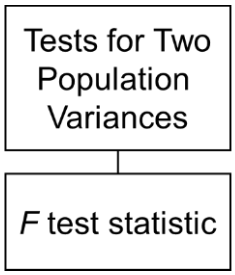
Has an *F* distribution with $(n_x - 1)$ numerator degrees of freedom and $(n_y - 1)$ denominator degrees of freedom

Denote an *F* value with v_1 numerator and v_2 denominator degrees of freedom by F_{v_1, v_2}

 Slide - 34

34

Test Statistic



The critical value for a hypothesis test about two population variances is

$$F = \frac{s_x^2}{s_y^2}$$

where F has $(n_x - 1)$ numerator degrees of freedom and $(n_y - 1)$ denominator degrees of freedom

 Slide - 35

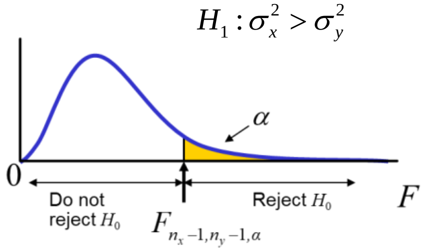
35

Decision Rules: Two Variances

Use s_x^2 to denote the larger variance.

$H_0: \sigma_x^2 \le \sigma_y^2$

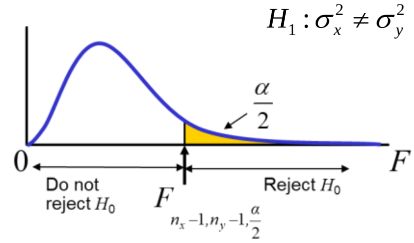
$H_1: \sigma_x^2 > \sigma_y^2$



Reject H_0 if $F > F_{n_x-1, n_y-1, \alpha}$

$H_0: \sigma_x^2 = \sigma_y^2$

$H_1: \sigma_x^2 \ne \sigma_y^2$



- rejection region for a two-tail test is:

Reject H_0 if $F > F_{n_x-1, n_y-1, \frac{\alpha}{2}}$

where s_x^2 is the larger of the two sample variances

 Slide - 36

36

Example 2: *F* Test

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
Mean	**3.27**	**2.53**
Std dev	**1.30**	**1.16**

Is there a difference in the variances between the NYSE NASDAQ at the $\alpha = 0.10$ level?

 Slide - 37

37

F Test: Example Solution (1 of 2)

- Form the hypothesis test:
  $H_0 : \sigma_x^2 = \sigma_y^2$ (there is no difference between variances)
  $H_1 : \sigma_x^2 \neq \sigma_y^2$ (there is a difference between variances)
- Find the *F* critical values for $\alpha = .\frac{10}{2}$:

Degrees of Freedom:

- Numerator (NYSE has the larger standard deviation):
  - $n_x - 1 = 21 - 1 = 20$ d.f.
- Denominator:
  - $n_y - 1 = 25 - 1 = 24$ d.f.

$$F_{n_x - 1, n_y - 1, \frac{\alpha}{2}}$$

$$= F_{20,24,0.\frac{10}{2}} = \boxed{2.03}$$

 Slide - 38

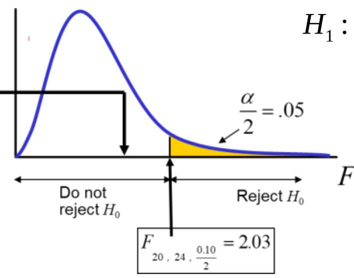
38

F Test: Example Solution (2 of 2)

- The test statistic is:

$$H_0: \sigma_x^2 = \sigma_y^2$$
$$H_1: \sigma_x^2 \neq \sigma_y^2$$

$$F = \frac{s_x^2}{s_y^2} = \frac{1.30^2}{1.16^2} = \boxed{1.256}$$



- *F* = 1.256 is not in the rejection region, so we do not reject H_0
- Conclusion: There is not sufficient evidence of a difference in variances at $\alpha = .10$

Some Comments on Hypothesis Testing

- A test with low power can result from:
  - Small sample size
  - Large variances in the underlying populations
  - Poor measurement procedures
- If sample sizes are large it is possible to find significant differences that are not practically important
- Researchers should select the appropriate level of significance before computing *p*-values

Two-Sample Tests in Excel

For paired samples (*t* test):

- Data | data analysis | t – test : paired two sample for means

For independent samples:

- Independent sample *z* test with variances known:
  - Data | data analysis | z – test : two sample for means

For variances…

- *F* test for two variances:
  - Data | data analysis | F – test : two sample for variances

Chapter Summary (1 of 2)

- Compared two dependent samples (paired samples)
  - Performed paired sample *t* test for the mean difference
- Compared two independent samples
  - Performed *z* test for the differences in two means
  - Performed pooled variance *t* test for the differences in two means
- Compared two population proportions
  - Performed *z*-test for two population proportions

Chapter Summary (2 of 2)

- Performed *F* tests for the difference between two population variances
- Used the *F* table to find *F* critical values