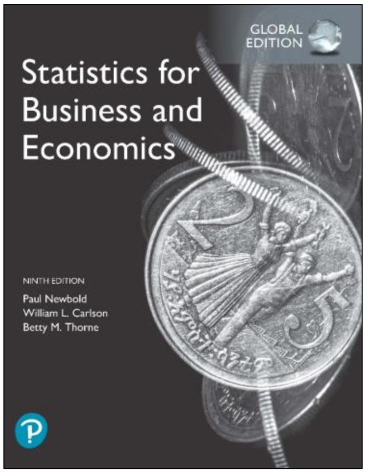
Statistics for Business and Economics

Ninth Edition, Global Edition



Chapter 12

Multiple Regression

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Chapter Goals

After completing this chapter, you should be able to:

- Apply multiple regression analysis to business decision-making situations
- Analyze and interpret the computer output for a multiple regression model
- Perform a hypothesis test for all regression coefficients or for a subset of coefficients
- Fit and interpret nonlinear regression models
- Incorporate qualitative variables into the regression model by using dummy variables
- Discuss model specification and analyze residuals

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Section 12.1 The Multiple Regression Model

Idea: Examine the linear relationship between
1 dependent (*Y*) & 2 or more independent variables (X_i)

Multiple Regression Model with *K* Independent Variables:

Y-intercept — Population slopes — Random Error

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_K X_K + \varepsilon$$

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Multiple Regression Equation

The coefficients of the multiple regression model are estimated using sample data

Multiple regression equation with *K* independent variables:

Estimated (or predicted) value of *y* — Estimated intercept — Estimated slope coefficients

$$\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \ldots + b_K x_{Ki}$$

In this chapter we will always use a computer to obtain the regression slope coefficients and other regression summary measures.

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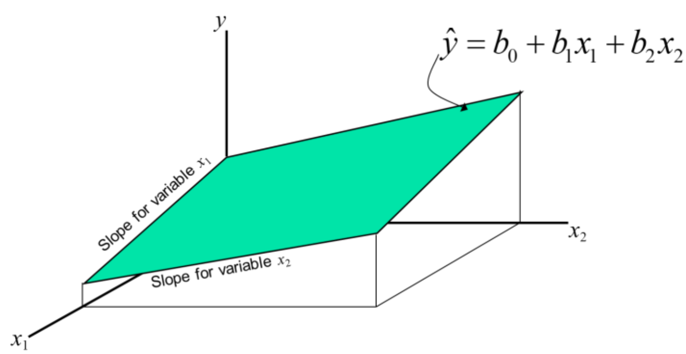
Three Dimensional Graphing (1 of 2)

Two variable model



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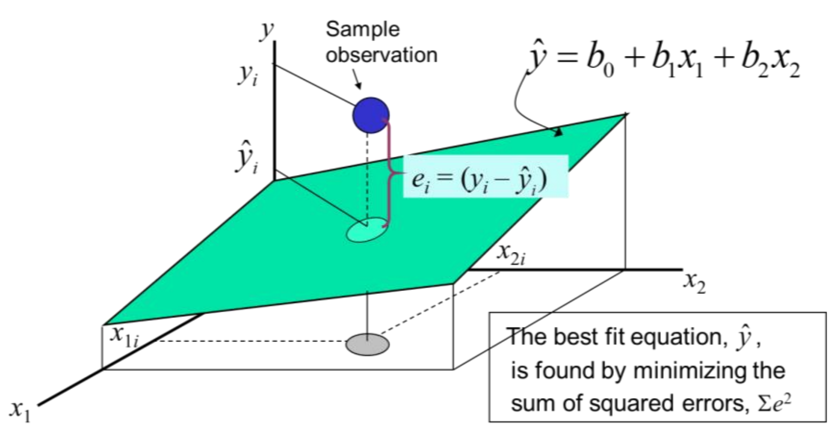
Three Dimensional Graphing (2 of 2)

Two variable model



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Section 12.2 Estimation of Coefficients

Standard Multiple Regression Assumptions

- 1. The x_{ji} terms are fixed numbers, or they are realizations of random variables X_j that are independent of the error terms, ε_i
- 2. The expected value of the random variable *Y* is a linear function of the independent X_j variables.
- 3. The error terms are normally distributed random variables with mean 0 and a constant variance, σ^2.

$$E\left[\varepsilon_i\right]=0 \text{ and } E\left[\varepsilon_i^2\right]=\sigma^2 \text{ for } \left(i=1,\ldots,n\right)$$

(The constant variance property is called homoscedasticity)

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Standard Multiple Regression Assumptions

- 4. The random error terms, ε_i, are not correlated with one another, so that

$$E\left[\varepsilon_i \varepsilon_j\right]=0 \text{ for all } i \neq j$$

- 5. It is not possible to find a set of numbers, $c_0, c_1, \ldots, c_k$, such that

$$c_0 + c_1 x_{1i} + c_2 x_{2i} + \ldots + c_K x_{Ki} = 0$$

(This is the property of no linear relation for the $X_j s$)

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Example 1: 2 Independent Variables

- A distributor of frozen desert pies wants to evaluate factors thought to influence demand
  - Dependent variable: Pie sales (units per week)
  - Independent variables: Price (in $); Advertising ($100's)
- Data are collected for 15 weeks

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Pie Sales Example

Week	Pie Sales	Price ($)	Advertising ($100s)
1	350	5.50	3.3
2	460	7.50	3.3
3	350	8.00	3.0
4	430	8.00	4.5
5	350	6.80	3.0
6	380	7.50	4.0
7	430	4.50	3.0
8	470	6.40	3.7
9	450	7.00	3.5
10	490	5.00	4.0
11	340	7.20	3.5
12	300	7.90	3.2
13	440	5.90	4.0
14	450	5.00	3.5
15	300	7.00	2.7

Multiple regression equation:

$$\widehat{\text{Sales}} = b_0 + b_1 \text{ (Price)} + b_2 \text{ (Advertising)}$$

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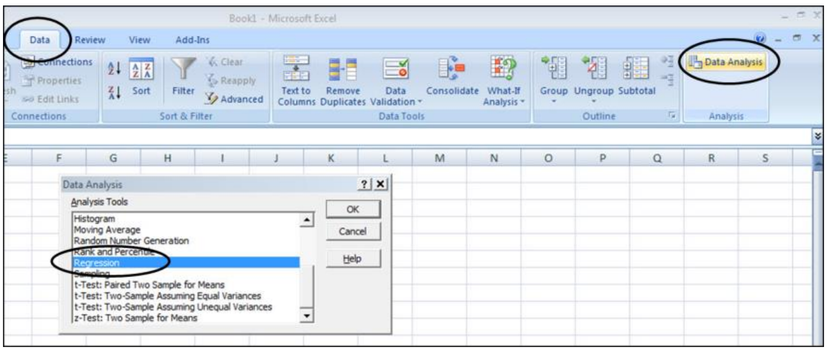
Estimating a Multiple Linear Regression Equation

- Excel can be used to generate the coefficients and measures of goodness of fit for multiple regression
  - Data / Data Analysis / Regression



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Multiple Regression Output

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$$\widehat{\text{Sales}} = 306.526 - 24.975(\text{Price}) + 74.131(\text{Advertising})$$

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
Total	14	56493.333			

	Coefficients	Standard Error	*t* Stat	*P*-value	Lower 95%	Upper 95%
Intercept	306.52619	114.25389	2.68285	0.01993	57.58835	555.46404
Price	−24.97509	10.83213	−2.30565	0.03979	−48.57626	−1.37392
Advertising	74.13096	25.96732	2.85478	0.01449	17.55303	130.70888

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The Multiple Regression Equation

$$\widehat{\text{Sales}} = 306.526 - 24.975(\text{Price}) + 74.131(\text{Advertising})$$

where
Sales is in number of pies per week
Price is in $
Advertising is in $100's.

$b_1 = -24.975$: sales will decrease, on average, by 24.975 pies per week for each $1 increase in selling price, net of the effects of changes due to advertising

$b_2 = 74.131$: sales will increase, on average, by 74.131 pies per week for each $100 increase in advertising, net of the effects of changes due to price

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Section 12.3 Explanatory Power of a Multiple Regression Equation

Coefficient of Determination, R^2

- Reports the proportion of total variation in *y* explained by all *x* variables taken together

$$R^2 = \frac{\text{SSR}}{\text{SST}} = \frac{\text{regression sum of squares}}{\text{total sum of squares}}$$

- This is the ratio of the explained variability to total sample variability

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Coefficient of Determination, *R* Squared

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$$R^2 = \frac{\text{SSR}}{\text{SST}} = \frac{29460.0}{56493.3} = .52148$$

52.1% of the variation in pie sales is explained by the variation in price and advertising

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
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Estimation of Error Variance

- Consider the population regression model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_K x_{Ki} + \varepsilon_i$$

- The unbiased estimate of the variance of the errors is

$$s_e^2 = \frac{\sum_{i=1}^{n} e_i^2}{n - K - 1} = \frac{\text{SSE}}{n - K - 1}$$

where $e_i = y_i - \hat{y}_i$

- The square root of the variance, s_e, is called the standard error of the estimate

Standard Error, *s* Sub Epsilon

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$s_e = 47.463$

The magnitude of this value can be compared to the average y value

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
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Adjusted Coefficient of Determination, *R* Bar Squared (1 of 2)

- R^2 never decreases when a new *X* variable is added to the model, even if the new variable is not an important predictor variable
  - This can be a disadvantage when comparing models
- What is the **net effect** of adding a new variable?
  - We lose a degree of freedom when a new *X* variable is added
  - Did the new *X* variable add enough explanatory power to offset the loss of one degree of freedom?

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Adjusted Coefficient of Determination, *R* Bar Squared (2 of 2)

- Used to correct for the fact that adding non-relevant independent variables will still reduce the error sum of squares

$$\bar{R}^2 = 1 - \frac{\text{SSE}/(n-K-1)}{\text{SST}/(n-1)}$$

(where *n* = sample size, *K* = number of independent variables)

  – Adjusted R^2 provides a better comparison between multiple regression models with different numbers of independent variables
  – Penalize excessive use of unimportant independent variables
  – Value is less than R^2

R Bar Squared

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$\bar{R}^2 = .44172$

44.2% of the variation in pie sales is explained by the variation in price and advertising, taking into account the sample size and number of independent variables

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
Total	14	56493.333			

	Coefficients	Standard Error	*t* Stat	*P*-value	Lower 95%	Upper 95%
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Coefficient of Multiple Correlation

- The coefficient of multiple correlation is the correlation between the predicted value and the observed value of the dependent variable

$$R = r\left(\hat{y}, y\right) = \sqrt{R^2}$$

- Is the square root of the multiple coefficient of determination
- Used as another measure of the strength of the linear relationship between the dependent variable and the independent variables
- Comparable to the correlation between *Y* and *X* in simple regression

Section 12.4 Conf. Intervals and Hypothesis Tests for Regression Coefficients

The variance of a coefficient estimate is affected by:

- the sample size
- the spread of the *X* variables
- the correlations between the independent variables, and
- the model error term

We are typically more interested in the regression coefficients b_j than in the constant or intercept b_0

Confidence Intervals (1 of 2)

Confidence interval limits for the population slope β_j

$$b_j \pm t_{n-K-1,\frac{\alpha}{2}} S_{b_j}$$

where *t* has (*n* − *K* − 1) d.f.

	Coefficients	Standard Error
Intercept	306.52619	114.25389
Price	−24.97509	10.83213
Advertising	74.13096	25.96732

Here, *t* has (15 − 2 − 1) = 12 d.f.

Example: Form a 95% confidence interval for the effect of changes in price (x_1) on pie sales:

$$-24.975 \pm (2.1788)(10.832)$$

So the interval is $-48.576 < \beta_1 < -1.374$

Confidence Intervals (2 of 2)

Confidence interval for the population slope β_i

	Coefficients	Standard Error	…	Lower 95%	Upper 95%
Intercept	306.52619	114.25389	…	57.58835	555.46404
Price	−24.97509	10.83213	…	−48.57626	−1.37392
Advertising	74.13096	25.96732	…	17.55303	130.70888

Example: Excel output also reports these interval endpoints:

Weekly sales are estimated to be reduced by between 1.37 to 48.58 pies for each increase of $1 in the selling price

Hypothesis Tests

- Use *t*-tests for individual coefficients
- Shows if a specific independent variable is conditionally important
- Hypotheses:
  - $H_0 : \beta_j = 0$ (no linear relationship)
  - $H_1 : \beta_j \neq 0$ (linear relationship does exist between x_j and *y*)

Evaluating Individual Regression Coefficients (1 of 3)

$H_0 : \beta_j = 0$ (no linear relationship)

$H_1 : \beta_j \neq 0$ (linear relationship does exist between x_i and *y*)

Test Statistic:

$$t = \frac{b_j - 0}{S_{b_j}} \qquad (\text{df} = n - k - 1)$$

Evaluating Individual Regression Coefficients (2 of 3)

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

***t*-value for Price is $t = -2.306$, with *p*-value .0398**

***t*-value for Advertising is $t = 2.855$, with *p*-value .0145**

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
Total	14	56493.333			

	Coefficients	Standard Error	*t* Stat	*P*-value	Lower 95%	Upper 95%
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Advertising	74.13096	25.96732	2.85478	0.01449	17.55303	130.70888

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Example 2: Evaluating Individual Regression Coefficients

$H_0: \beta_j = 0$

$H_1: \beta_j \neq 0$

$\text{d.f.} = 15 - 2 - 1 = 12$

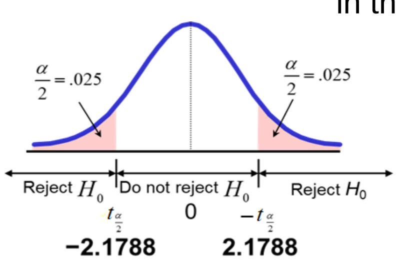
$\alpha = .05$

$t_{12,.025} = 2.1788$

From Excel output:

	Coefficients	Standard Error	*t* Stat	*P*-value
Price	−24.97509	10.83213	−2.30565	0.03979
Advertising	74.13096	25.96732	2.85478	0.01449

The test statistic for each variable falls in the rejection region (*p*-values < .05)



Decision:

Reject H_0 for each variable

Conclusion:

There is evidence that both Price and Advertising affect pie sales at $\alpha = .05$

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Section 12.5 Tests on Regression Coefficients

Tests on All Coefficients

- *F*-Test for Overall Significance of the Model
- Shows if there is a linear relationship between all of the *X* variables considered together and *Y*
- Use *F* test statistic
- Hypotheses:

$H_0 : \beta_1 = \beta_2 = ... = \beta_K = 0$ (no linear relationship)

H_1 : at least one $\beta_i \neq 0$ (at least one independent variable affects *Y*)

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F-Test for Overall Significance (1 of 3)

- Test statistic:

$$F = \frac{\text{MSR}}{s_e^2} = \frac{\text{SSR}/K}{\text{SSE}/(n-K-1)}$$

where *F* has *K* (numerator) and ($n - K - 1$) (denominator) degrees of freedom

- The decision rule is

$$\text{Reject } H_0 \text{ if } F = \frac{\text{MSR}}{s_e^2} > F_{K,n-K-1,\alpha}$$

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F-Test for Overall Significance (2 of 3)

Regression Statistics	
Multiple R	0.72213
R Square	0.52148
Adjusted R Square	0.44172
Standard Error	47.46341
Observations	15

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{14730.0}{2252.8} = 6.5386$$

With 2 and 12 degrees of freedom

P-value for the F-Test

ANOVA	*df*	*SS*	*MS*	*F*	Significance *F*
Regression	2	29460.027	14730.013	6.53861	0.01201
Residual	12	27033.306	2252.776		
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	Coefficients	Standard Error	*t* Stat	*P*-value	Lower 95%	Upper 95%
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Advertising	74.13096	25.96732	2.85478	0.01449	17.55303	130.70888

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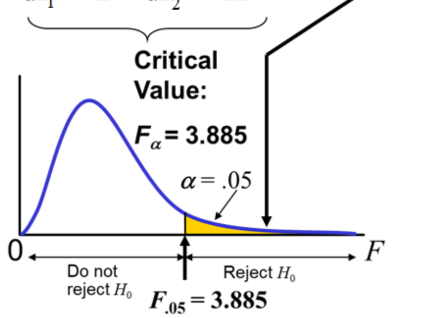
F-Test for Overall Significance (3 of 3)

$H_0 : \beta_1 = \beta_2 = 0$

$H_1 : \beta_1$ and β_2 not both zero

$\alpha = .05$

$\text{df}_1 = 2 \quad \text{df}_2 = 12$



Test Statistic:

$$F = \frac{\text{MSR}}{\text{MSE}} = 6.5386$$

Decision:

Since *F* test statistic is in the rejection region (p-value < .05), reject H_0

Conclusion:

There is evidence that at least one independent variable affects *Y*

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Test on a Subset of Regression Coefficients (1 of 2)

- Consider a multiple regression model involving variables X_j and Z_j, and the null hypothesis that the *Z* variable coefficients are all zero:

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_K x_K + \alpha_1 z_1 + \cdots \alpha_R z_R + \varepsilon$$

$$H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_R = 0$$

$$H_1 : \text{at least one of } \alpha_j \neq 0 \ (j = 1, \ldots, R)$$

Test on a Subset of Regression Coefficients (2 of 2)

- Goal: compare the error sum of squares for the complete model with the error sum of squares for the restricted model
  - First run a regression for the complete model and obtain SSE
  - Next run a restricted regression that excludes the *Z* variables (the number of variables excluded is *R*) and obtain the restricted error sum of squares SSE(*R*)
  - Compute the *F* statistic and apply the decision rule for a significance level α

$$\text{Reject } H_0 \text{ if } F = \frac{\left(\text{SSE}(\text{R}) - \text{SSE}\right)/\text{R}}{s_e^2} > F_{R, n-K-R-1, \alpha}$$

Section 12.6 Prediction

- Given a population regression model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_K x_{Ki} + \varepsilon_i \ \left(i = 1, 2, \ldots, n\right)$$

- then given a new observation of a data point

$$\left(x_{1,n+1,}\ x_{2,n+1}, \ldots,\ x_{K,n+1}\right)$$

the best linear unbiased forecast of $\hat{y}_{n+1}$ is

$$\hat{y}_{n+1} = b_0 + b_1 x_{1,n+1} + b_2 x_{2,n+1} + \cdots + b_K x_{K,n+1}$$

- It is risky to forecast for new *X* values outside the range of the data used to estimate the model coefficients, because we do not have data to support that the linear model extends beyond the observed range.

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Predictions from a Multiple Regression Model

Predict sales for a week in which the selling price is \$5.50 and advertising is \$350:

$$\begin{aligned}\widehat{\text{Sales}} &= 306.526 - 24.975(\text{Price}) + 74.131(\text{Advertising}) \\ &= 306.526 - 24.975(5.50) + 74.131(3.5) \\ &= 428.62\end{aligned}$$

Predicted sales is 428.62 pies

Note that Advertising is in \$100's, so \$350 means that $X_2 = 3.5$

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Section 12.7 Transformations for Nonlinear Regression Models

- The relationship between the dependent variable and an independent variable may not be linear
- Can review the scatter diagram to check for non-linear relationships
- Example: Quadratic model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \varepsilon$$

  - The second independent variable is the square of the first variable

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Quadratic Model Transformations

Quadratic model form:

Let $z_1 = x_1$ and $z_2 = x_1^2$

And specify the model as

$$y_i = \beta_0 + \beta_1 z_{1i} + \beta_2 z_{2i} + \varepsilon_i$$

- where:

$\beta_0 =$ *Y* intercept

$\beta_1 =$ regression coefficient for linear effect of *X* on *Y*

$\beta_2 =$ regression coefficient for quadratic effect on *Y*

$\varepsilon_i =$ random error in *Y* for observation *i*

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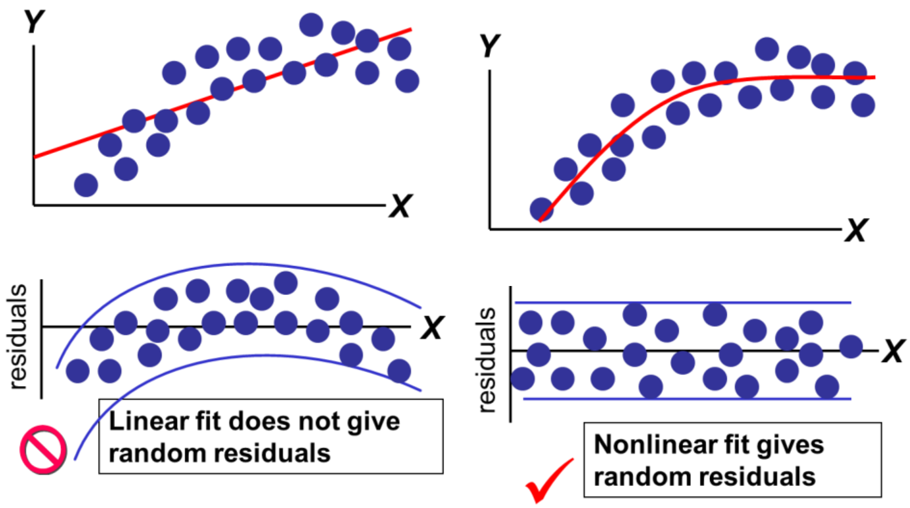
Linear vs. Nonlinear Fit



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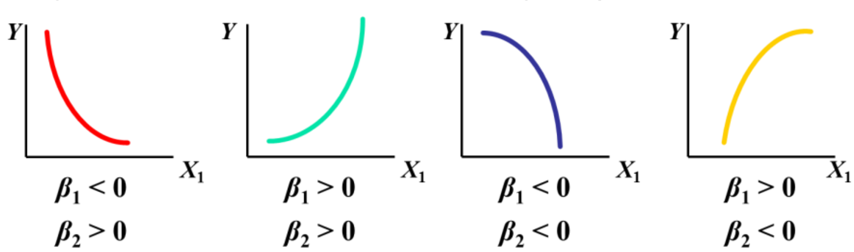
Quadratic Regression Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{1i}^2 + \varepsilon_i$$

Quadratic models may be considered when the scatter diagram takes on one of the following shapes:



β_1 = the coefficient of the linear term

β_2 = the coefficient of the squared term

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Testing for Significance: Quadratic Effect (1 of 3)

- Testing the Quadratic Effect
  - Compare the linear regression estimate

$$\hat{y} = b_0 + b_1 x_1$$

  - with quadratic regression estimate

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_1^2$$

  - Hypotheses
    - $H_0 : \beta_2 = 0$ (The quadratic term does not improve the model)
    - $H_1 : \beta_2 \neq 0$ (The quadratic term improves the model)

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Testing for Significance: Quadratic Effect (2 of 3)

- Testing the Quadratic Effect

  Hypotheses
  - $H_0 : \beta_2 = 0$ (The quadratic term does not improve the model)
  - $H_1 : \beta_2 \neq 0$ (The quadratic term improves the model)

- The test statistic is

$$t = \frac{b_2 - \beta_2}{S_{b_2}}$$

$$\text{d.f} = n - 3$$

where:

$b_2 =$ squared term slope coefficient

$\beta_2 =$ hypothesized slope (zero)

$S_{b_2} =$ standard error of the slope

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Testing for Significance: Quadratic Effect (3 of 3)

- Testing the Quadratic Effect

Compare R^2 from simple regression to $\bar{R}^2$ from the quadratic model

- If $\bar{R}^2$ from the quadratic model is larger than R^2 from the simple model, then the quadratic model is a better model

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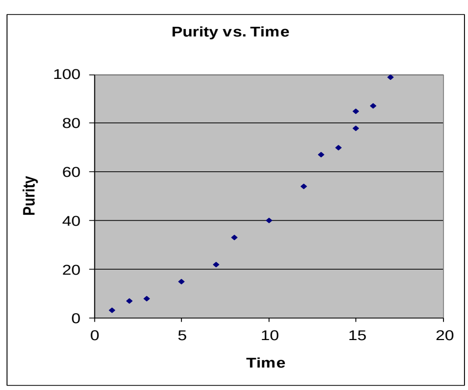
Example 3: Quadratic Model (1 of 3)

Purity	Filter Time
3	1
7	2
8	3
15	5
22	7
33	8
40	10
54	12
67	13
70	14
78	15
85	15
87	16
99	17

- Purity increases as filter time increases:



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Example 3: Quadratic Model (2 of 3)

- Simple regression results:

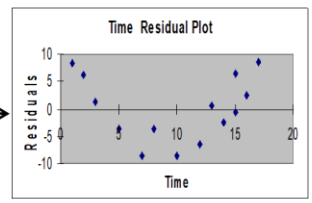
$$\hat{y} = -11.283 + 5.985\ \text{Time}$$

	Coefficients	Standard Error	*t* Stat	*P*-value
Intercept	−11.28267	3.46805	−3.25332	0.00691
Time	5.98520	0.30966	**19.32819**	2.078E-10

Regression Statistics	
R Square	**0.96888**
Adjusted R Square	0.96628
Standard Error	**6.15997**

F	Significance *F*
373.57904	2.0778E-10

t statistic, *F* statistic, and R^2 are all high, but the residuals are not random: →



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Example 3: Quadratic Model (3 of 3)

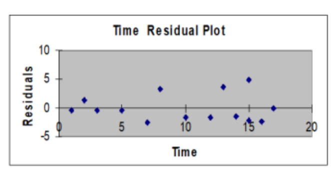
- Quadratic regression results:

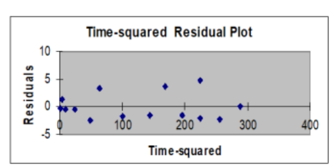
$$\hat{y} = 1.539 + 1.565\ \text{Time} + 0.245\ (\text{Time})^2$$

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.53870	2.24465	0.68550	0.50722
Time	1.56496	0.60179	2.60052	0.02467
Time-squared	0.24516	0.03258	**7.52406**	1.165E-05

Regression Statistics	
R Square	0.99494
Adjusted R Square	**0.99402**
Standard Error	**2.59513**

F	Significance *F*
1080.7330	2.368E-13





The quadratic term is significant and improves the model: $\bar{R}^2$ is higher and s_e is lower, residuals are now random

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Logarithmic Transformations

The Exponential Model:

- Original exponential model

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \varepsilon$$

- Transformed logarithmic model

$$\log(Y) = \log(\beta_0) + \beta_1 \log(X_1) + \beta_2 \log(X_2) + \log(\varepsilon)$$

Interpretation of coefficients

For the logarithmic model:

$$\log Y_i = \log \beta_0 + \beta_1 \log X_{1i} + \log \varepsilon_i$$

- When both dependent and independent variables are logged:
  - The estimated coefficient b_k of the independent variable X_k can be interpreted as

    a 1 percent change in X_k leads to an estimated b_k percentage change in the average value of *Y*
  - b_k is the elasticity of *Y* with respect to a change in X_k

Section 12.8 Dummy Variables for Regression Models

- A dummy variable is a categorical independent variable with two levels:
  - yes or no, on or off, male or female
  - recorded as 0 or 1
- Regression intercepts are different if the variable is significant
- Assumes equal slopes for other variables
- If more than two levels, the number of dummy variables needed is (number of levels - 1)

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Dummy Variable Example (1 of 2)

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2$$

Let:

y = Pie Sales

x_1 = Price

x_2 = Holiday ($x_2 = 1$ if a holiday occurred during the week)
($x_2 = 0$ if there was no holiday that week)

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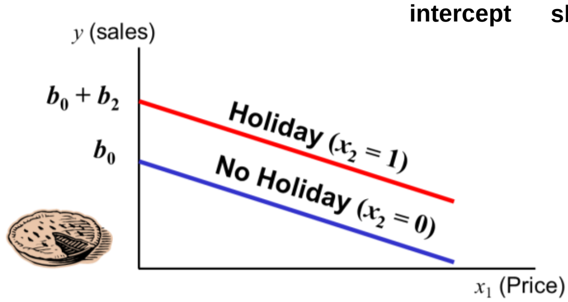
Dummy Variable Example (2 of 2)

$$\hat{y} = b_0 + b_1 x_1 + b_2(1) = (b_0 + b_2) + b_1 x_1 \quad \text{Holiday}$$

$$\hat{y} = b_0 + b_1 x_1 + b_2(0) = \quad b_0 \quad + b_1 x_1 \quad \text{No Holiday}$$

Different intercept **Same slope**



If $H_0 : \beta_2 = 0$ is rejected, then "Holiday" has a significant effect on pie sales

Interpreting the Dummy Variable Coefficient

Example: Sales = 300 − 30(Price) + 15(Holiday)

Sales: number of pies sold per week

Price: pie price in $

$$\text{Holiday}: \begin{cases} 1 \text{ If a holiday occurred during the week} \\ 0 \text{ If no holiday occurred} \end{cases}$$

$b_2 = 15$: on average, sales were 15 pies greater in weeks with a holiday than in weeks without a holiday, given the same price

Differences in Slope

- Hypothesizes interaction between pairs of *x* variables
  - Response to one *x* variable may vary at different levels of another *x* variable
- Contains two-way cross product terms
  - $$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3$$
    $$= b_0 + b_1 x_1 + b_2 x_2 + b_3 (x_1 x_2)$$

Effect of Interaction

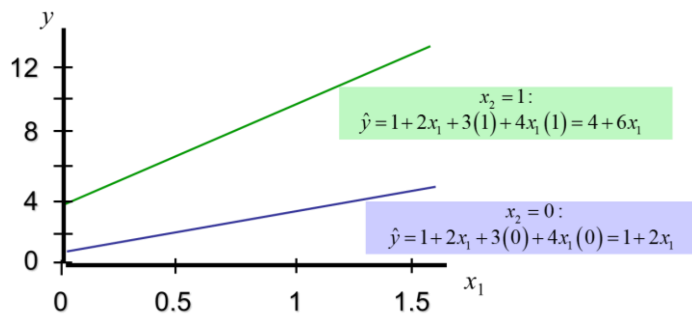
- Given:

$$Y = \beta_0 + \beta_2 X_2 + (\beta_1 + \beta_3 X_2) X_1$$
$$= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$$

- Without interaction term, effect of X_1 on *Y* is measured by β_1
- With interaction term, effect of X_1 on *Y* is measured by $\beta_1 + \beta_3 X_2$
- Effect changes as X_2 changes

Interaction Example

Suppose x_2 is a dummy variable and the estimated regression equation is $\hat{y} = 1 + 2x_1 + 3x_2 + 4x_1x_2$



Slopes are different if the effect of x_1 on *y* depends on x_2 value

Significance of Interaction Term

- The coefficient b_3 is an estimate of the difference in the coefficient of x_1 when $x_2 = 1$ compared to when $x_2 = 0$
- The *t* statistic for b_3 can be used to test the hypothesis

$$H_0 : \beta_3 = 0 | \beta_1 \neq 0,\ \beta_2 \neq 0$$
$$H_1 : \beta_3 \neq 0 | \beta_1 \neq 0,\ \beta_2 \neq 0$$

- If we reject the null hypothesis we conclude that there is a difference in the slope coefficient for the two subgroups

Section 12.9 Multiple Regression Analysis Application Procedure

Errors (residuals) from the regression model:

$$e_i = (y_i - \hat{y}_i)$$

Assumptions:

- The errors are normally distributed
- Errors have a constant variance
- The model errors are independent

Analysis of Residuals

- These residual plots are used in multiple regression:
  - Residuals vs. $\hat{y}_i$
  - Residuals vs. x_{1i}
  - Residuals vs. x_{2i}
  - Residuals vs. time (if time series data)

Use the residual plots to check for violations of regression assumptions

Chapter Summary

- Developed the multiple regression model
- Tested the significance of the multiple regression model
- Discussed adjusted $R^2 (\overline{R}^2)$
- Tested individual regression coefficients
- Tested portions of the regression model
- Used quadratic terms and log transformations in regression models
- Explained dummy variables
- Evaluated interaction effects
- Discussed using residual plots to check model assumptions

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