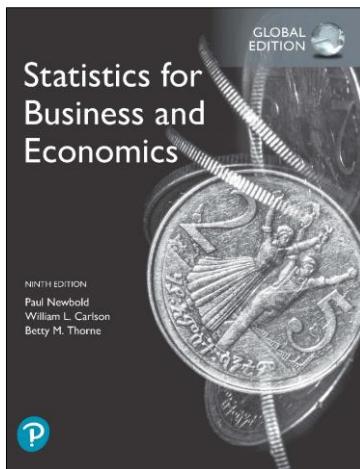


# Statistics for Business and Economics

Ninth Edition, Global Edition



## Chapter 5

### Continuous Random Variables and Probability Distributions

 Pearson

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## Chapter Goals (1 of 2)

**After completing this chapter, you should be able to:**

- Explain the difference between a discrete and a continuous random variable
- Describe the characteristics of the uniform and normal distributions
- Translate normal distribution problems into standardized normal distribution problems
- Find probabilities using a normal distribution table

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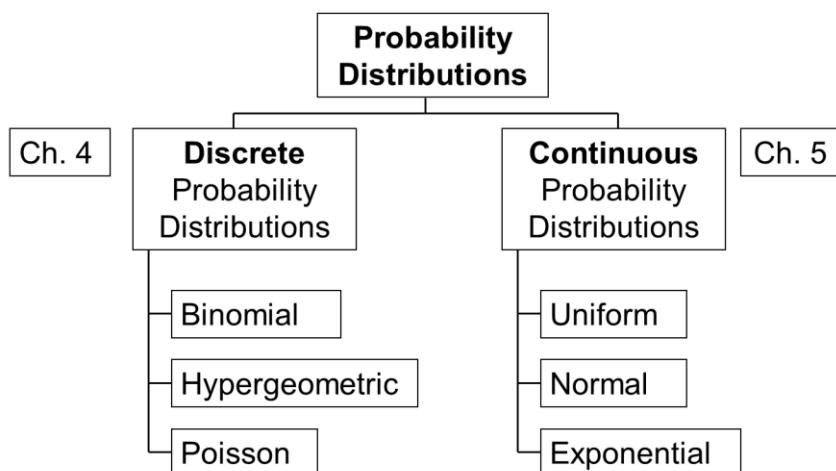
Slide - 2

## Chapter Goals (2 of 2)

**After completing this chapter, you should be able to:**

- Evaluate the normality assumption
- Use the normal approximation to the binomial distribution
- Recognize when to apply the exponential distribution
- Explain jointly distributed variables and linear combinations of random variables
- Explain examples to Financial Investment Portfolios

## Probability Distributions



## Section 5.1 Continuous Random Variables

- A continuous random variable is a variable that can assume any value in an interval
  - thickness of an item
  - time required to complete a task
  - temperature of a solution
  - height, in inches
- These can potentially take on any value, depending only on the ability to measure accurately.

## Cumulative Distribution Function

- The cumulative distribution function,  $F(x)$ , for a continuous random variable  $X$  expresses the probability that  $X$  does not exceed the value of  $x$

$$F(x) = P(X \leq x)$$

- Let  $a$  and  $b$  be two possible values of  $X$ , with  $a < b$ . The probability that  $X$  lies between  $a$  and  $b$  is

$$P(a < X < b) = F(b) - F(a)$$

## Probability Density Function (1 of 2)

The probability density function,  $f(x)$ , of random variable  $X$  has the following properties:

1.  $f(x) > 0$  for all values of  $x$
2. The area under the probability density function  $f(x)$  over all values of the random variable  $X$  within its range, is equal to 1.0
3. The probability that  $X$  lies between two values is the area under the density function graph between the two values

$$P(a < X < b) = \int_a^b f(x) dx$$

## Probability Density Function (2 of 2)

The probability density function,  $f(x)$ , of random variable  $X$  has the following properties:

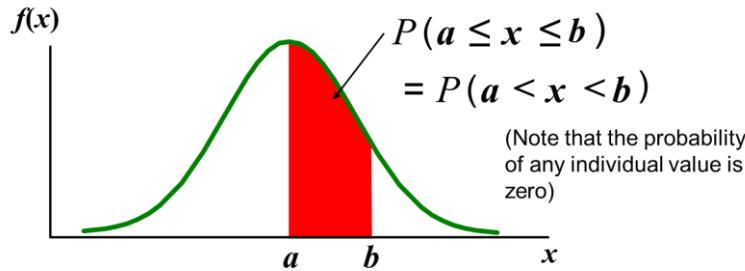
4. The cumulative density function  $F(x_0)$ , is the area under the probability density function  $f(x)$  from the minimum  $x$  value up to  $x_0$

$$F(x_0) = \int_{x_m}^{x_0} f(x) dx$$

where  $x_m$  is the minimum value of the random variable  $x$

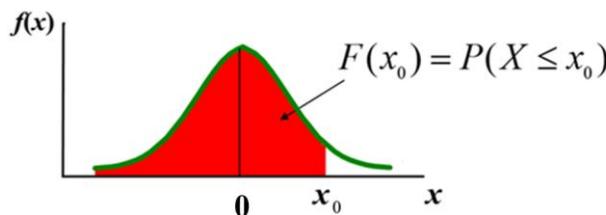
## Probability as an Area (1 of 2)

Shaded area under the curve is the probability that  $X$  is between  $a$  and  $b$

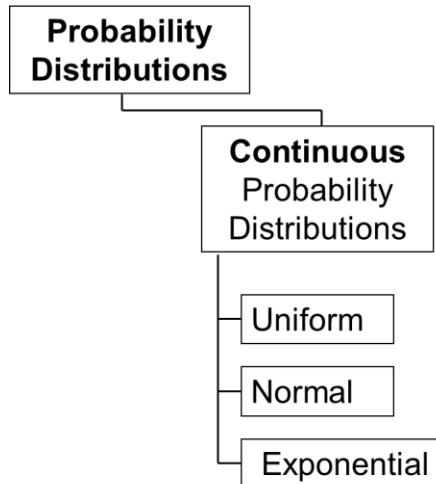


## Probability as an Area (2 of 2)

1. The total area under the curve  $f(x)$  is 1
2. The area under the curve  $f(x)$  to the left of  $x_0$  is  $F(x_0)$ , where  $x_0$  is any value that the random variable can take.

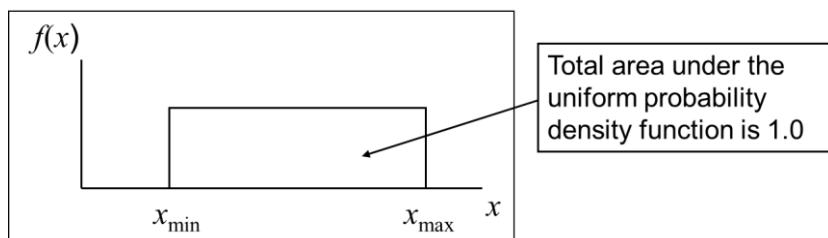


## The Uniform Distribution (1 of 3)



## The Uniform Distribution (2 of 3)

- The uniform distribution is a probability distribution that has equal probabilities for all equal-width intervals within the range of the random variable



## The Uniform Distribution (3 of 3)

The Continuous Uniform Distribution:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

where

$f(x)$  = value of the density function at any  $x$  value

$a$  = minimum value of  $x$

$b$  = maximum value of  $x$



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## Section 5.2 Expectations for Continuous Random Variables

- The mean of  $X$ , denoted  $\mu_X$ , is defined as the expected value of  $X$

$$\mu_X = E[X]$$

- The variance of  $X$ , denoted  $\sigma_X^2$ , is defined as the expectation of the squared deviation,  $(X - \mu_X)^2$ , of a random variable from its mean

$$\sigma_X^2 = E[(X - \mu_X)^2]$$



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## Mean and Variance of the Uniform Distribution

- The mean of a uniform distribution is

$$\mu = \frac{a+b}{2}$$

- The variance is

$$\sigma^2 = \frac{(b-a)^2}{12}$$

Where  $a$  = minimum value of  $x$

$b$  = maximum value of  $x$



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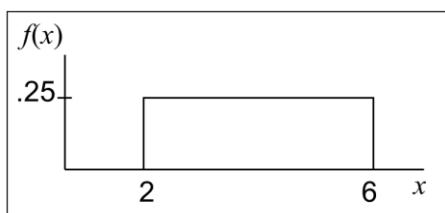
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## Uniform Distribution Example

Example: Uniform probability distribution over the range  $2 \leq x \leq 6$ :

$$f(x) = \frac{1}{6-2} = .25 \text{ for } 2 \leq x \leq 6$$



$$\mu = \frac{a+b}{2} = \frac{2+6}{2} = 4$$

$$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(6-2)^2}{12} = 1.333$$



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## Linear Functions of Random Variables (1 of 2)

- Let  $W = a + bX$ , where  $X$  has mean  $\mu_X$  and variance  $\sigma_X^2$ , and  $a$  and  $b$  are constants
- Then the mean of  $W$  is

$$\mu_W = E[a + bX] = a + b\mu_X$$

- the variance is

$$\sigma_W^2 = \text{Var}[a + bX] = b^2\sigma_X^2$$

- the standard deviation of  $W$  is

$$\sigma_W = |b|\sigma_X$$

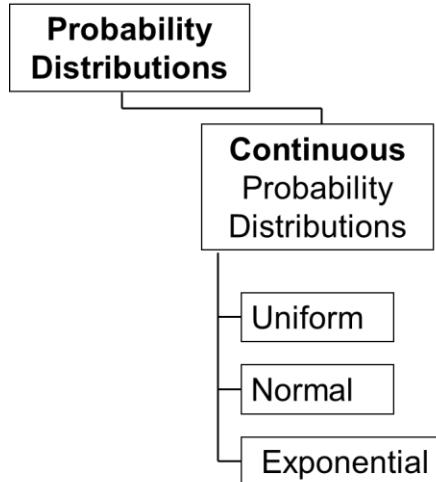
## Linear Functions of Random Variables (2 of 2)

- An important special case of the result for the linear function of random variable is the standardized random variable

$$Z = \frac{X - \mu_X}{\sigma_X}$$

- which has a mean 0 and variance 1

## Section 5.3 The Normal Distribution (1 of 3)



## Section 5.3 The Normal Distribution (2 of 3)

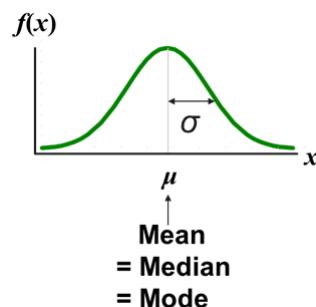
- Bell Shaped
- Symmetrical
- Mean, Median and Mode are Equal

Location is determined by the mean,  $\mu$

Spread is determined by the standard deviation,  $\sigma$

The random variable has an infinite theoretical range:

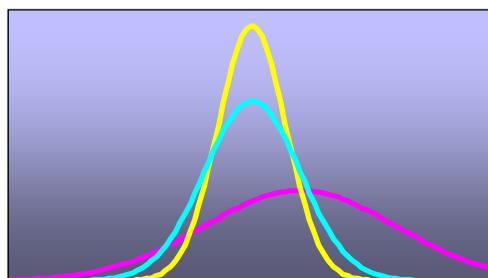
$+\infty$  to  $-\infty$



## Section 5.3 The Normal Distribution (3 of 3)

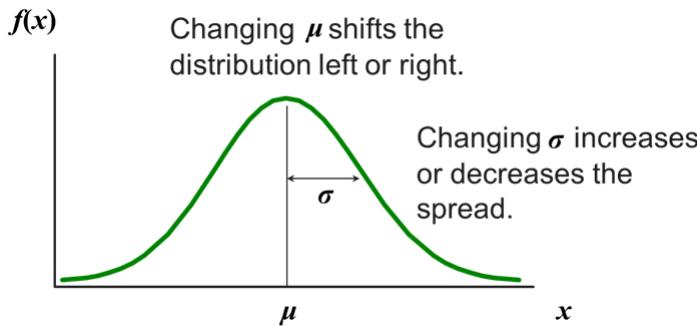
- The normal distribution closely approximates the probability distributions of a wide range of random variables
- Distributions of sample means approach a normal distribution given a “large” sample size
- Computations of probabilities are direct and elegant
- The normal probability distribution has led to good business decisions for a number of applications

## Many Normal Distributions



By varying the parameters  $\mu$  and  $\sigma$ , we obtain different normal distributions

## The Normal Distribution Shape



Given the mean  $\mu$  and variance  $\sigma^2$  we define the normal distribution using the notation

$$X \sim N(\mu, \sigma^2)$$

## The Normal Probability Density Function

- The formula for the normal probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where  $e$  = the mathematical constant approximated by 2.71828

$\pi$  = the mathematical constant approximated by 3.14159

$\mu$  = the population mean

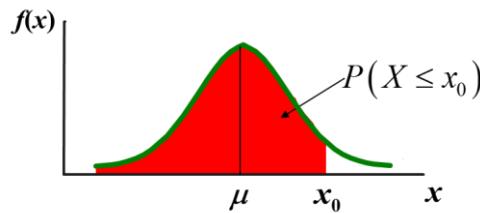
$\sigma^2$  = the population variance

$x$  = any value of the continuous variable,  $-\infty < x < \infty$

## Cumulative Normal Distribution

- For a normal random variable  $X$  with mean  $\mu$  and variance  $\sigma^2$ , i.e.,  $X \sim N(\mu, \sigma^2)$ , the cumulative distribution function is

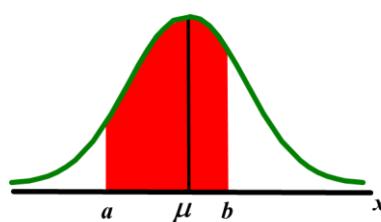
$$F(x_0) = P(X \leq x_0)$$



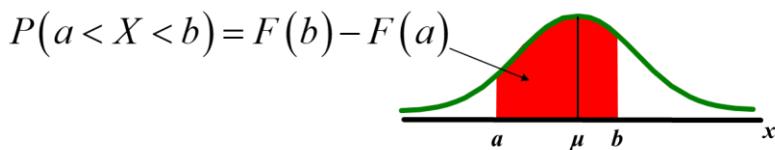
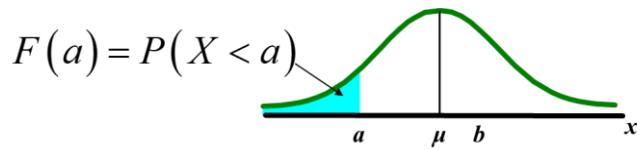
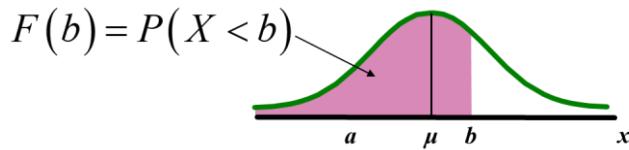
## Finding Normal Probabilities (1 of 5)

The probability for a range of values is measured by the area under the curve

$$P(a < X < b) = F(b) - F(a)$$



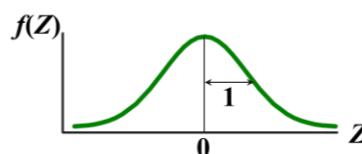
## Finding Normal Probabilities (2 of 5)



## The Standard Normal Distribution

- Any normal distribution (with any mean and variance combination) can be transformed into the standardized normal distribution ( $Z$ ), with mean 0 and variance 1

$$Z \sim N(0,1)$$



- Need to transform  $X$  units into  $Z$  units by subtracting the mean of  $X$  and dividing by its standard deviation

$$Z = \frac{X - \mu}{\sigma}$$

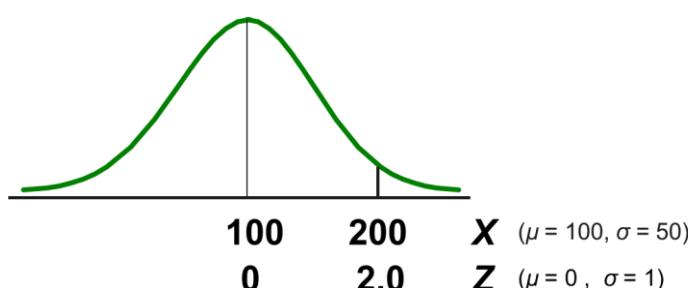
## Example 1

- If  $X$  is distributed normally with mean of 100 and standard deviation of 50, the  $Z$  value for  $X = 200$  is

$$Z = \frac{X - \mu}{\sigma} = \frac{200 - 100}{50} = 2.0$$

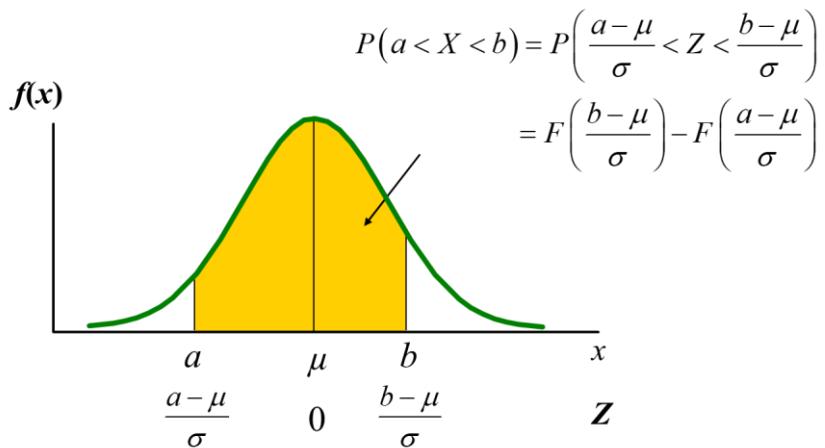
- This says that  $X = 200$  is two standard deviations (2 increments of 50 units) above the mean of 100.

## Comparing $X$ and $Z$ Units



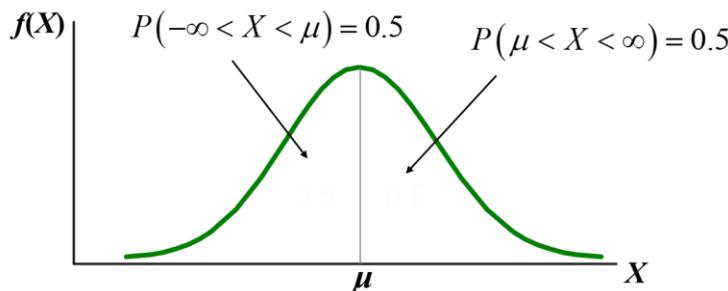
Note that the distribution is the same, only the scale has changed. We can express the problem in original units ( $X$ ) or in standardized units ( $Z$ )

## Finding Normal Probabilities (3 of 5)



## Probability as Area Under the Curve

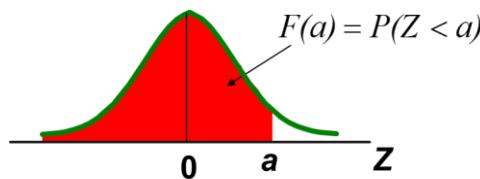
The total area under the curve is 1.0, and the curve is symmetric, so half is above the mean, half is below



$$P(-\infty < X < \infty) = 1.0$$

## Appendix Table 1

- The Standard Normal Distribution table in the textbook (Appendix Table 1) shows values of the cumulative normal distribution function
- For a given  $Z$ -value  $a$ , the table shows  $F(a)$  (the area under the curve from negative infinity to  $a$ )

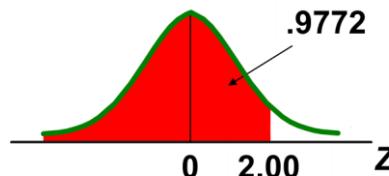


## The Standard Normal Table (1 of 2)

- Appendix Table 1 gives the probability  $F(a)$  for any value  $a$

Example:

$$P(Z < 2.00) = .9772$$

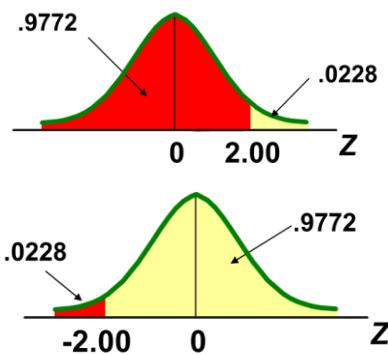


## The Standard Normal Table (2 of 2)

- For negative  $Z$ -values, use the fact that the distribution is symmetric to find the needed probability:

Example:

$$\begin{aligned} P(Z < -2.00) &= 1 - 0.9772 \\ &= 0.0228 \end{aligned}$$



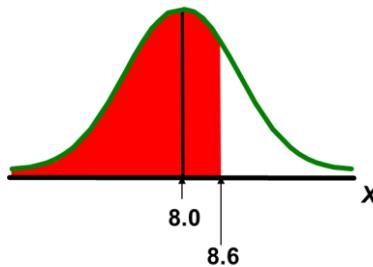
## General Procedure for Finding Probabilities

To find  $P(a < X < b)$  when  $X$  is distributed normally:

- Draw the normal curve for the problem in terms of  $X$
- Translate  $X$ -values to  $Z$ -values
- Use the Cumulative Normal Table

## Finding Normal Probabilities (4 of 5)

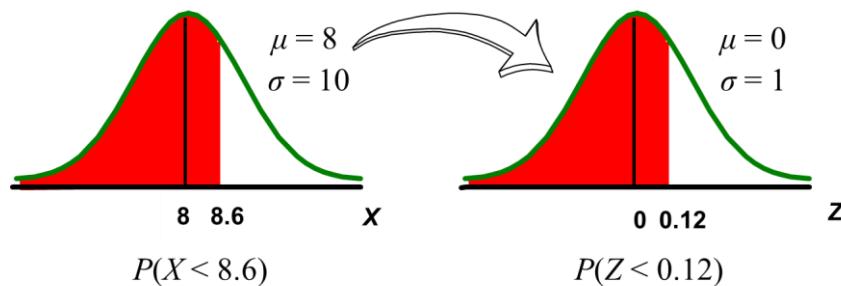
- Suppose  $X$  is normal with mean 8.0 and standard deviation 5.0
- Find  $P(X < 8.6)$



## Finding Normal Probabilities (5 of 5)

- Suppose  $X$  is normal with mean 8.0 and standard deviation 5.0. Find  $P(X < 8.6)$

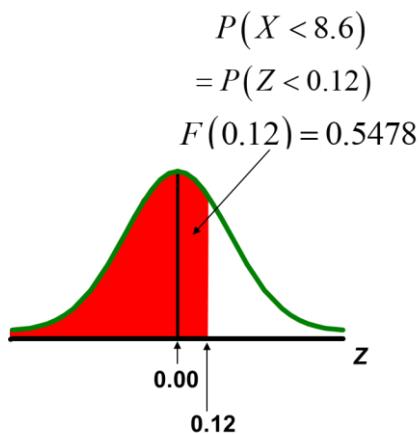
$$Z = \frac{X - \mu}{\sigma} = \frac{8.6 - 8.0}{5.0} = 0.12$$



## Solution: Finding $P$ Left Parenthesis $Z$ Is Less Than 0.12 Right Parenthesis

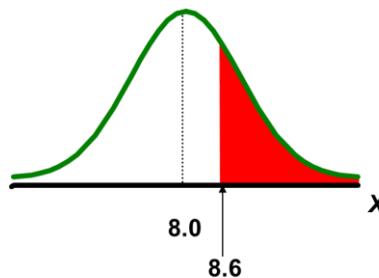
Standardized Normal Probability Table (Portion)

$z$	$F(z)$
.10	.5398
.11	.5438
.12	.5478
.13	.5517



## Upper Tail Probabilities (1 of 2)

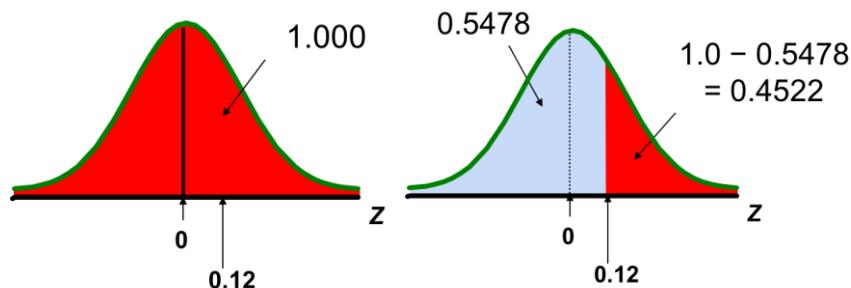
- Suppose  $X$  is normal with mean 8.0 and standard deviation 5.0.
- Now Find  $P(X > 8.6)$



## Upper Tail Probabilities (2 of 2)

- Now Find  $P(X > 8.6)$ ...

$$\begin{aligned} P(X > 8.6) &= P(Z > 0.12) = 1.0 - P(Z \leq 0.12) \\ &= 1.0 - 0.5478 = 0.4522 \end{aligned}$$



## Finding the $X$ Value for a Known Probability (1 of 2)

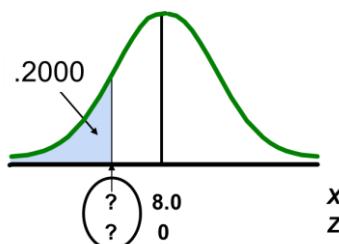
- Steps to find the  $X$  value for a known probability:
  - Find the  $Z$  value for the known probability
  - Convert to  $X$  units using the formula:

$$X = \mu + Z\sigma$$

## Finding the $X$ Value for a Known Probability (2 of 2)

Example:

- Suppose  $X$  is normal with mean 8.0 and standard deviation 5.0.
- Now find the  $X$  value so that only 20% of all values are below this  $X$



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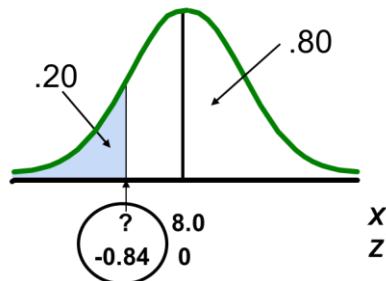
## Find the $Z$ Value for 20% in the Lower Tail

- Find the  $Z$  value for the known probability

Standardized Normal Probability Table (Portion)

- 20% area in the lower tail is consistent with a  $Z$  value of -0.84

$z$	$F(z)$
.82	.7939
.83	.7967
.84	.7995
.85	.8023



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## Finding the $X$ Value

2. Convert to  $X$  units using the formula:

$$\begin{aligned}X &= \mu + Z\sigma \\&= 8.0 + (-0.84)5.0 \\&= 3.80\end{aligned}$$

So 20% of the values from a distribution with mean 8.0 and standard deviation 5.0 are less than 3.80

## Assessing Normality

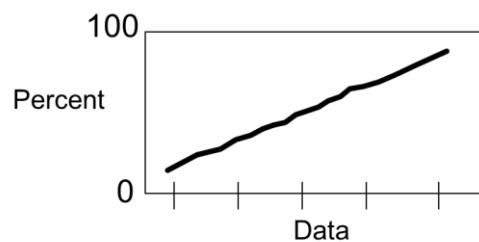
- Not all continuous random variables are normally distributed
- It is important to evaluate how well the data is approximated by a normal distribution

## The Normal Probability Plot (1 of 3)

- Normal probability plot
  - Arrange data from low to high values
  - Find cumulative normal probabilities for all values
  - Examine a plot of the observed values vs. cumulative probabilities (with the cumulative normal probability on the vertical axis and the observed data values on the horizontal axis)
  - Evaluate the plot for evidence of linearity

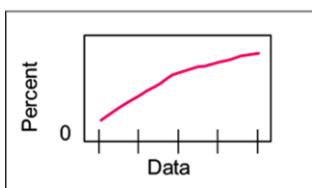
## The Normal Probability Plot (2 of 3)

A normal probability plot for data from a normal distribution will be approximately linear:

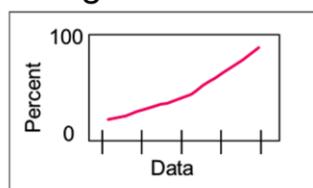


## The Normal Probability Plot (3 of 3)

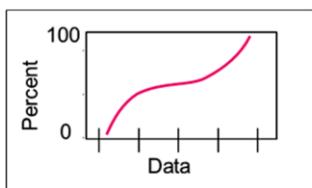
Left-Skewed



Right-Skewed



Uniform



Nonlinear plots indicate a deviation from normality

## Section 5.4 Normal Distribution Approximation for Binomial Distribution (1 of 3)

- Recall the binomial distribution:
  - $n$  independent trials
  - probability of success on any given trial =  $P$
- Random variable  $X$ :
  - $X_i = 1$  if the  $i^{\text{th}}$  trial is “success”
  - $X_i = 0$  if the  $i^{\text{th}}$  trial is “failure”

$$E[X] = \mu = nP$$

$$\text{Var}(X) = \sigma^2 = nP(1-P)$$

## Section 5.4 Normal Distribution Approximation for Binomial Distribution (2 of 3)

- The shape of the binomial distribution is approximately normal if  $n$  is large
- The normal is a good approximation to the binomial when  $nP(1-P) > 5$
- Standardize to  $Z$  from a binomial distribution:

$$Z = \frac{X - E[X]}{\sqrt{\text{Var}(X)}} = \frac{X - np}{\sqrt{nP(1-P)}}$$

## Section 5.4 Normal Distribution Approximation for Binomial Distribution (3 of 3)

- Let  $X$  be the number of successes from  $n$  independent trials, each with probability of success  $P$ .
- If  $nP(1-P) > 5$ ,

$$P(a < X < b) = P\left(\frac{a - np}{\sqrt{nP(1-P)}} \leq Z \leq \frac{b - np}{\sqrt{nP(1-P)}}\right)$$

## Binomial Approximation Example

- 40% of all voters support ballot proposition A. What is the probability that between 76 and 80 voters indicate support in a sample of  $n = 200$ ?

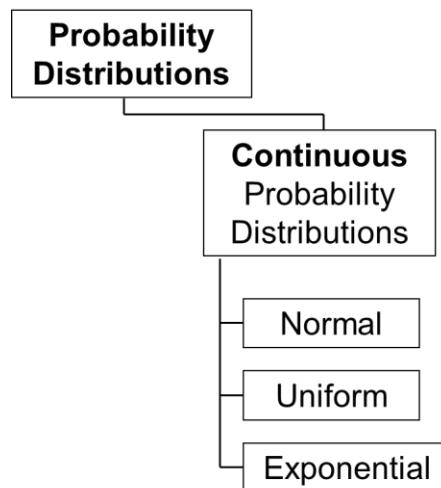
—  $E[X] = \mu = nP = 200(0.40) = 80$

—  $Var(X) = \sigma^2 = nP(1-P) = 200(0.40)(1-0.40) = 48$

(note:  $nP(1-P) = 48 > 5$ )

$$\begin{aligned} P(76 < X < 80) &= P\left(\frac{76-80}{\sqrt{200(0.4)(1-0.4)}} \leq Z \leq \frac{80-80}{\sqrt{200(0.4)(1-0.4)}}\right) \\ &= P(-0.58 < Z < 0) \\ &= F(0) - F(-0.58) \\ &= 0.5000 - 0.2810 = 0.2190 \end{aligned}$$

## Section 5.5 The Exponential Distribution (1 of 4)



## Section 5.5 The Exponential Distribution (2 of 4)

- Used to model the length of time between two occurrences of an event (the time between arrivals)
  - Examples:
    - Time between trucks arriving at an unloading dock
    - Time between transactions at an ATM Machine
    - Time between phone calls to the main operator

## Section 5.5 The Exponential Distribution (3 of 4)

- The exponential random variable  $T (t > 0)$  has a probability density function

$$f(t) = \lambda e^{-\lambda t} \text{ for } t > 0$$

- Where
  - $\lambda$  is the mean number of occurrences per unit time
  - $t$  is the number of time units until the next occurrence
  - $e = 2.71828$
- $T$  is said to follow an exponential probability distribution

## Section 5.5 The Exponential Distribution (4 of 4)

- Defined by a single parameter, its mean  $\lambda$  (lambda)
- The cumulative distribution function (the probability that an arrival time is less than some specified time  $t$ ) is

$$F(t) = 1 - e^{-\lambda t}$$

where  $e$  = mathematical constant approximated by 2.71828

$\lambda$  = the population mean number of arrivals per unit

$t$  = any value of the continuous variable where  $t > 0$

## Exponential Distribution Example

Example: Customers arrive at the service counter at the rate of 15 per hour. What is the probability that the arrival time between consecutive customers is less than three minutes?

- The mean number of arrivals per hour is 15, so  $\lambda = 15$
- Three minutes is .05 hours
- $P(\text{arrival time} < .05) = 1 - e^{-\lambda x} = 1 - e^{-(15)(.05)} = 0.5276$
- So there is a 52.76% probability that the arrival time between successive customers is less than three minutes

## Section 5.6 Jointly Distributed Continuous Random Variables (1 of 2)

- Let  $X_1, X_2, \dots, X_k$  be continuous random variables
- Their joint cumulative distribution function,

$$F(x_1, x_2, \dots, x_k)$$

defines the probability that simultaneously  $X_1$  is less than  $x_1$ ,  $X_2$  is less than  $x_2$ , and so on; that is

$$F(x_1, x_2, \dots, x_k) = P(X_1 < x_1 \cap X_2 < x_2 \cap \dots \cap X_k < x_k)$$

## Section 5.6 Jointly Distributed Continuous Random Variables (2 of 2)

- The cumulative distribution functions

$$F(x_1), F(x_2), \dots, F(x_k)$$

of the individual random variables are called their marginal distribution functions

- The random variables are independent if and only if

$$F(x_1, x_2, \dots, x_k) = F(x_1)F(x_2) \cdots F(x_k)$$

## Covariance

- Let  $X$  and  $Y$  be continuous random variables, with means  $\mu_x$  and  $\mu_y$
- The expected value of  $(X - \mu_x)(Y - \mu_y)$  is called the covariance between  $X$  and  $Y$

$$Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)]$$

- An alternative but equivalent expression is

$$Cov(X, Y) = E[XY] - \mu_x \mu_y$$

- If the random variables  $X$  and  $Y$  are independent, then the covariance between them is 0. However, the converse is not true.

## Correlation

- Let  $X$  and  $Y$  be jointly distributed random variables.
- The correlation between  $X$  and  $Y$  is

$$\rho = Corr(X, Y) = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

## Sums of Random Variables (1 of 2)

Let  $X_1, X_2, \dots, X_k$  be  $k$  random variables with means  $\mu_1, \mu_2, \dots, \mu_k$  and variances  $\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2$ . Then:

- The mean of their sum is the sum of their means

$$E[(X_1 + X_2 + \dots + X_k)] = \mu_1 + \mu_2 + \dots + \mu_k$$

## Sums of Random Variables (2 of 2)

Let  $X_1, X_2, \dots, X_k$  be  $k$  random variables with means  $\mu_1, \mu_2, \dots, \mu_k$  and variances  $\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2$ . Then:

- If the covariance between every pair of these random variables is 0, then the variance of their sum is the sum of their variances

$$Var(X_1 + X_2 + \dots + X_k) = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_k^2$$

- However, if the covariances between pairs of random variables are not 0, the variance of their sum is

$$Var(X_1 + X_2 + \dots + X_k) = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_k^2 + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^k Cov(X_i, X_j)$$

## Differences Between a Pair of Random Variables

For two random variables,  $X$  and  $Y$

- The mean of their difference is the difference of their means; that is

$$E[X - Y] = \mu_X - \mu_Y$$

- If the covariance between  $X$  and  $Y$  is 0, then the variance of their difference is

$$Var[X - Y] = \sigma_X^2 + \sigma_Y^2$$

- If the covariance between  $X$  and  $Y$  is not 0, then the variance of their difference is

$$Var[X - Y] = \sigma_X^2 + \sigma_Y^2 - 2Cov(X, Y)$$

## Linear Combinations of Random Variables (1 of 2)

- A linear combination of two random variables,  $X$  and  $Y$ , (where  $a$  and  $b$  are constants) is

$$W = aX + bY$$

- The mean of  $W$  is

$$\mu_W = E[W] = E[aX + bY] = a\mu_X + b\mu_Y$$

## Linear Combinations of Random Variables (2 of 2)

- The variance of  $W$  is

$$\sigma_w^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\text{Cov}(X, Y)$$

- Or using the correlation,

$$\sigma_w^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\rho(X, Y)\sigma_x\sigma_y$$

- If both  $X$  and  $Y$  are joint normally distributed random variables then the linear combination,  $W$ , is also normally distributed

## Example 2 (1 of 2)

- Two tasks must be performed by the same worker.
  - $X$  = minutes to complete task 1;  $\mu_x = 20$ ,  $\sigma_x = 5$
  - $Y$  = minutes to complete task 2;  $\mu_y = 20$ ,  $\sigma_y = 5$
  - $X$  and  $Y$  are normally distributed and independent
- What is the mean and standard deviation of the time to complete both tasks?

## Example 2 (2 of 2)

- $X$  = minutes to complete task 1;  $\mu_x = 20$ ,  $\sigma_x = 5$
- $Y$  = minutes to complete task 2;  $\mu_y = 30$ ,  $\sigma_y = 8$
- What are the mean and standard deviation for the time to complete both tasks?

$$W = X + Y$$

$$\mu_W = \mu_X + \mu_Y = 20 + 30 = 50$$

- Since  $X$  and  $Y$  are independent,  $\text{Cov}(X, Y) = 0$ , so

$$\sigma_W^2 = \sigma_X^2 + \sigma_Y^2 + 2\text{Cov}(X, Y) = (5)^2 + (8)^2 = 89$$

- The standard deviation is

$$\sigma_w = \sqrt{89} = 9.434$$

## Financial Investment Portfolios

- A financial portfolio can be viewed as a linear combination of separate financial instruments

$$\begin{aligned} \left( \text{Return on} \right)_{\text{portfolio}} &= \left( \text{Proportion of} \right)_{\text{portfolio value}} \times \left( \text{Stock 1} \right)_{\text{return}} + \left( \text{Proportion of} \right)_{\text{portfolio value}} \times \left( \text{Stock 2} \right)_{\text{return}} \\ &\quad \dots + \left( \text{Proportion of} \right)_{\text{portfolio value}} \times \left( \text{Stock } N \right)_{\text{return}} \end{aligned}$$

## Portfolio Analysis Example (1 of 3)

- Consider two stocks,  $A$  and  $B$ 
  - The price of Stock  $A$  is normally distributed with mean 12 and variance 4
  - The price of Stock  $B$  is normally distributed with mean 20 and variance 16
  - The stock prices have a positive correlation,  $\rho_{AB} = .50$
- Suppose you own
  - 10 shares of Stock  $A$
  - 30 shares of Stock  $B$

## Portfolio Analysis Example (2 of 3)

- The mean and variance of this stock portfolio are:  
(Let  $W$  denote the distribution of portfolio value)

$$\mu_W = 10\mu_A + 20\mu_B = (10)(12) + (30)(20) = 720$$

$$\begin{aligned}\sigma_W^2 &= 10^2 \sigma_A^2 + 30^2 \sigma_B^2 + (2)(10)(30) \text{Corr}(A, B) \sigma_A \sigma_B \\ &= 10^2 (4)^2 + 30^2 (16)^2 + (2)(10)(30)(.50)(4)(16) \\ &= 251,200\end{aligned}$$

## Portfolio Analysis Example (3 of 3)

- What is the probability that your portfolio value is less than \$500?

$$\mu_w = 720$$

$$\sigma_w = \sqrt{251,200} = 501.20$$

- The  $Z$  value for 500 is  $Z = \frac{500 - 720}{501.20} = -0.44$
- $P(Z < -0.44) = 0.3300$ 
  - So the probability is 0.33 that your portfolio value is less than \$500.

## Chapter Summary

- Defined continuous random variables
- Presented key continuous probability distributions and their properties
  - uniform, normal, exponential
- Found probabilities using formulas and tables
- Interpreted normal probability plots
- Examined when to apply different distributions
- Applied the normal approximation to the binomial distribution
- Reviewed properties of jointly distributed continuous random variables