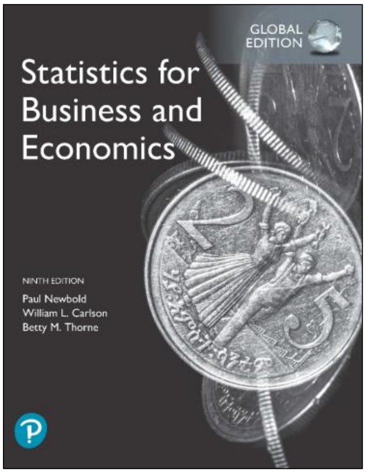
# Statistics for Business and Economics

Ninth Edition, Global Edition




## Chapter 8

Estimation:
Additional Topics

 Slide - 1

1

## Chapter Goals

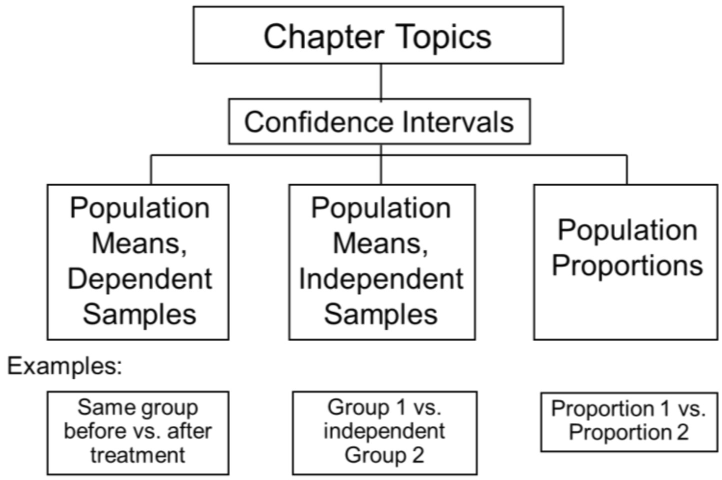
**After completing this chapter, you should be able to:**

- Form confidence intervals for the difference between two means from dependent samples
- Form confidence intervals for the difference between two independent population means (standard deviations known or unknown)
- Compute confidence interval limits for the difference between two independent population proportions

 Slide - 2

2

# Estimation: Additional Topics




 Slide - 3

3

# Section 8.1 Dependent Samples

Dependent samples

Confidence Interval Estimation of the Difference Between Two Normal Population Means: Dependent Samples

Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$d_i = x_i - y_i$$

- Eliminates Variation Among Subjects
- Assumptions:
  - Both Populations Are Normally Distributed

 Slide - 4

4

# Mean Difference

Dependent samples

The $i^{\text{th}}$ paired difference is $d_i$, where

$$d_i = x_i - y_i$$

The point estimate for the population mean paired difference is $\bar{d}$ :

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

The sample standard deviation is:

$$s_d = \sqrt{\frac{\sum_{i=1}^{n} \left(d_i - \bar{d}\right)^2}{n-1}}$$

*n* is the number of matched pairs in the sample

Pearson  Slide - 5

5

# Confidence Interval for Mean Difference (1 of 2)

Dependent samples

The confidence interval for the difference between two population means, $\mu_d$, is

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$$

Where

*n* = the sample size

(number of matched pairs in the paired sample)

Pearson  Slide - 6

6

# Confidence Interval for Mean Difference (2 of 2)

Dependent samples

- The margin of error is

$$ME = t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$$

- $t_{n-1,\frac{\alpha}{2}}$ is the value from the Student's *t* distribution with $(n-1)$ degrees of freedom for which

$$p\left(t_{n-1} > t_{n-1,\frac{\alpha}{2}}\right) = \frac{\alpha}{2}$$



7

# Paired Samples Example (1 of 2)

Dependent samples

- Six people sign up for a weight loss program. You collect the following data:

| | Weight: | | |
|---|---|---|---|
| Person | Before ($x$) | After ($y$) | Difference, $d_i$ |
| 1 | 136 | 125 | 11 |
| 2 | 205 | 195 | 10 |
| 3 | 157 | 150 | 7 |
| 4 | 138 | 140 | − 2 |
| 5 | 175 | 165 | 10 |
| 6 | 166 | 160 | 6 |
| | | | 42 |

$$\bar{d} = \frac{\sum d_i}{n}$$

$$= 7.0$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$$

$$= 4.82$$



8

# Paired Samples Example (2 of 2)

Dependent samples

- For a 95% confidence level, the appropriate *t* value is
  $t_{n-1,\frac{\alpha}{2}} = t_{5,.025} = 2.571$
- The 95% confidence interval for the difference between means, $\mu_d$, is

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{S_d}{\sqrt{n}}$$

$$7 \pm (2.571) \frac{4.82}{\sqrt{6}}$$

$$-1.94 < \mu_d < 12.06$$

Since this interval contains zero, we cannot be 95% confident, given this limited data, that the weight loss program helps people lose weight




9

# Section 8.2 Difference Between Two Means: Independent Samples

Population means, independent samples

Confidence Interval Estimation of the Difference Between Two Normal Population Means: Independent Samples

Goal: Form a confidence interval for the difference between two population means, $\mu_x - \mu_y$




10

# Difference Between Two Means: Independent Samples (1 of 2)

Population means, independent samples

Goal: Form a confidence interval for the difference between two population means, $\mu_x - \mu_y$

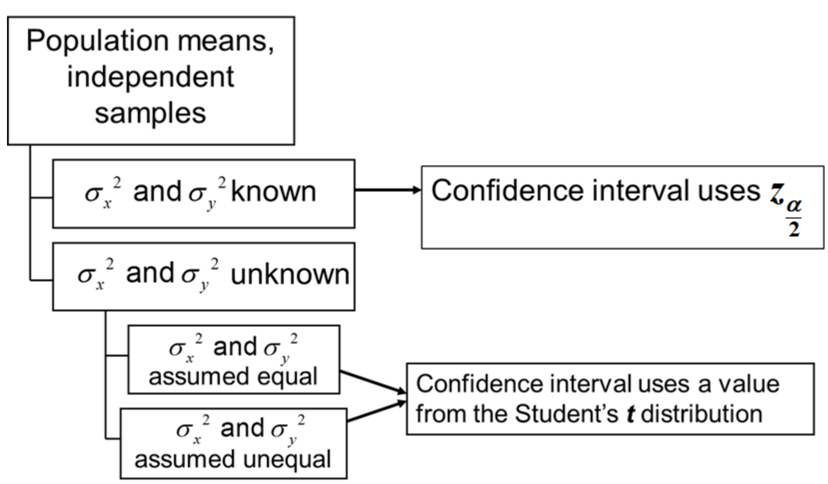
- Different data sources
  - Unrelated
  - Independent
    - Sample selected from one population has no effect on the sample selected from the other population
- The point estimate is the difference between the two sample means:

$$\bar{x} - \bar{y}$$

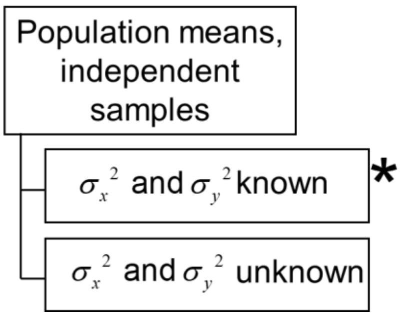


# Difference Between Two Means: Independent Samples (2 of 2)



# Sigma Sub *x* Squared and Sigma Sub *y* Squared Known (1 of 2)



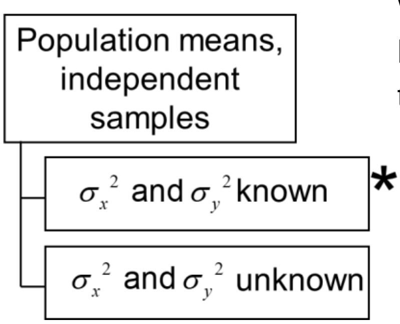

Assumptions:

- Samples are randomly and independently drawn
- both population distributions are normal
- Population variances are known

 Slide - 13

13

# Sigma Sub *x* Squared and Sigma Sub *y* Squared Known (2 of 2)




When $\sigma_x$ and $\sigma_y$ are known and both populations are normal, the variance of $\bar{X} - \bar{Y}$ is

$$\sigma^2_{\bar{X}-\bar{Y}} = \frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}$$

…and the random variable

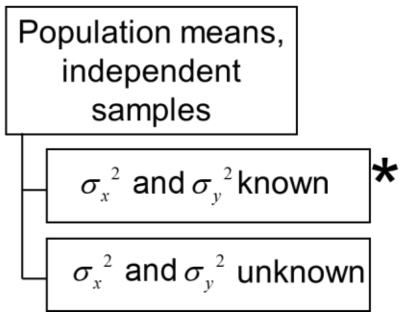
$$Z = \frac{(\bar{x} - \bar{y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_x^2}{n_X} + \frac{\sigma_y^2}{n_Y}}}$$

has a standard normal distribution

 Slide - 14

14

## Confidence Interval, Sigma Sub *x* Squared and Sigma Sub *y* Squared Known



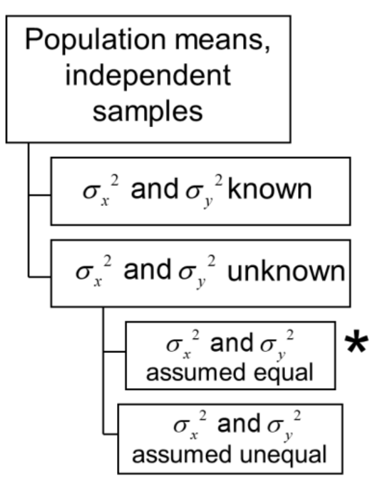

The confidence interval for $\mu_x - \mu_y$ is:

$$(\bar{x} - \bar{y}) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$$

 Slide - 15

15

## Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Equal (1 of 3)



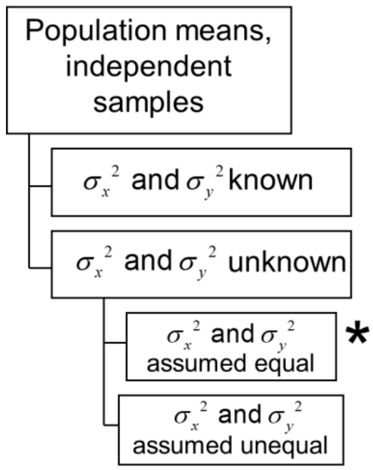

Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown but assumed equal

 Slide - 16

16

## Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Equal (2 of 3)

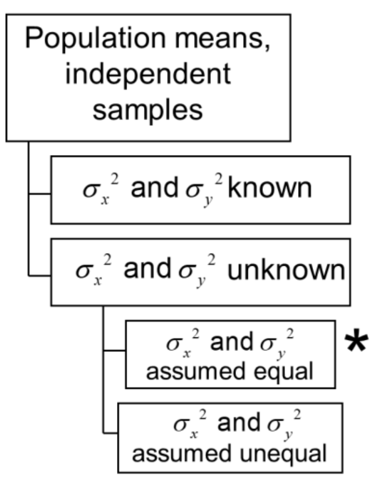



Forming interval estimates:

- The population variances are assumed equal, so use the two sample standard deviations and pool them to estimate $\sigma$
- use a *t* value with $(n_x + n_y - 2)$ degrees of freedom

## Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Equal (3 of 3)

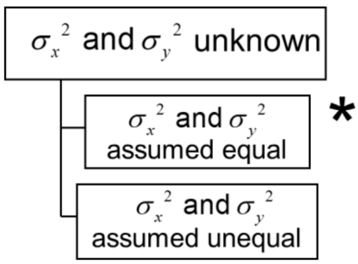



The pooled variance is

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

## Confidence Interval, Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Equal




The confidence interval for $\mu_1 - \mu_2$ is:

$$(\bar{x} - \bar{y}) \pm t_{n_x + n_y - 2, \frac{\alpha}{2}} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$$

Where $$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

 Slide - 19

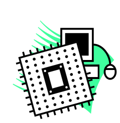
19

## Pooled Variance Example

You are testing two computer processors for speed. Form a confidence interval for the difference in CPU speed. You collect the following speed data (in Mhz):

| | $CPU_x$ | $CPU_y$ |
|---|---|---|
| **Number Tested** | **17** | **14** |
| **Sample mean** | **3004** | **2538** |
| **Sample std dev** | **74** | **56** |

Assume both populations are normal with equal variances, and use 95% confidence



 Slide - 20

20

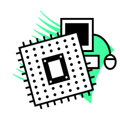
## Calculating the Pooled Variance

The pooled variance is:

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{(n_x - 1) + (n_y - 1)} = \frac{(17-1)74^2 + (14-1)56^2}{(17-1)+(14-1)} = 4427.03$$

The *t* value for a 95% confidence interval is:

$$t_{n_x + n_y - 2, \frac{\alpha}{2}} = t_{29, 0.025} = 2.045$$









## Calculating the Confidence Limits

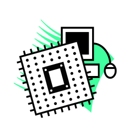
- The 95% confidence interval is

$$(\bar{x} - \bar{y}) \pm t_{n_x + n_y - 2, \frac{\alpha}{2}} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$$

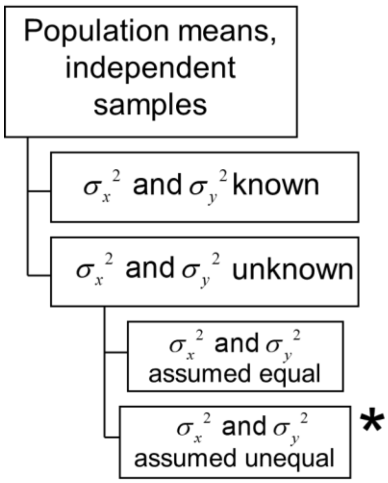
$$(3004 - 2538) \pm (2.054)\sqrt{\frac{4427.03}{17} + \frac{4427.03}{14}}$$

$$416.69 < \mu_X - \mu_Y < 515.31$$

We are 95% confident that the mean difference in CPU speed is between 416.69 and 515.31 Mhz.



## Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Unequal (1 of 2)



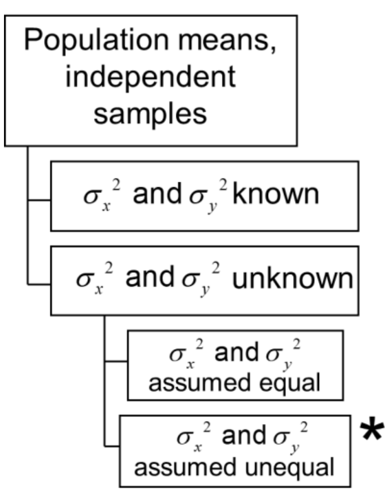

Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed
- Population variances are unknown and assumed unequal

 Slide - 23

23

## Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Assumed Unequal (2 of 2)




Forming interval estimates:

- The population variances are assumed unequal, so a pooled variance is not appropriate
- use a $t$ value with $\nu$ degrees of freedom, where

$$\nu = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x - 1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{(n_y - 1)}}$$

 Slide - 24

24

## Confidence Interval, Sigma Sub *x* Squared and Sigma Sub *y* Squared Unknown, Unequal

- $\sigma_x^2$ and $\sigma_y^2$ unknown
  - $\sigma_x^2$ and $\sigma_y^2$ assumed equal
  - $\sigma_x^2$ and $\sigma_y^2$ assumed unequal *

The confidence interval for $\mu_1 - \mu_2$ is:

$$(\bar{x} - \bar{y}) \pm t_{\nu, \frac{\alpha}{2}} \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$$

Where

$$\nu = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x - 1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{(n_y - 1)}}$$



## Section 8.3 Two Population Proportions (1 of 2)

Population proportions

Confidence Interval Estimation of the Difference Between Two Population Proportions (Large Samples)

Goal: Form a confidence interval for the difference between two population proportions, $P_x - P_y$

## Section 8.3 Two Population Proportions (2 of 2)

Population proportions

Goal: Form a confidence interval for the difference between two population proportions, $P_x - P_y$

Assumptions:

Both sample sizes are large (generally at least 40 observations in each sample)

The point estimate for the difference is $\hat{p}_x - \hat{p}_y$




## Two Population Proportions

Population proportions

- The random variable

$$Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}}}$$

is approximately normally distributed

## Confidence Interval for Two Population Proportions

Population proportions

The confidence limits for

$P_x - P_y$ are:

$$\left(\hat{p}_x - \hat{p}_y\right) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_x\left(1-\hat{p}_x\right)}{n_x} + \frac{\hat{p}_y\left(1-\hat{p}_y\right)}{n_y}}$$



## Example: Two Population Proportions (1 of 3)

Form a 90% confidence interval for the difference between the proportion of men and the proportion of women who have college degrees.

- In a random sample, 26 of 50 men and 28 of 40 women had an earned college degree

## Example: Two Population Proportions (2 of 3)

Men: $\hat{p}_x = \dfrac{26}{50} = 0.52$

Women: $\hat{p}_y = \dfrac{28}{40} = 0.70$

$$\sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}} = \sqrt{\frac{0.52(0.48)}{50} + \frac{0.70(0.30)}{40}} = 0.1012$$

For 90% confidence, $Z_{\frac{\alpha}{2}} = 1.645$

 Slide - 31

31

## Example: Two Population Proportions (3 of 3)

The confidence limits are:

$$\left(\hat{p}_x - \hat{p}_y\right) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}}$$

$$= (.52 - .70) \pm 1.645(0.1012)$$

so the confidence interval is

$$-0.3465 < P_x - P_y < -0.0135$$

Since this interval does not contain zero we are 90% confident that the two proportions are not equal

 Slide - 32

32

## Chapter Summary

- Compared two dependent samples (paired samples)
  - Formed confidence intervals for the paired difference
- Compared two independent samples
  - Formed confidence intervals for the difference between two means, population variance known, using $z$
  - Formed confidence intervals for the differences between two means, population variance unknown, using $t$
- Formed confidence intervals for the differences between two population proportions