1) The White Company is a member of the lamp industry, which is perfectly competitive. The price of a lamp is $50. The firm's total cost function is

$TC = 1{,}000 + 20Q + 5Q^2$

where TC is total cost (in dollars) and Q is hourly output.

a. What output maximizes profit?
b. What is the firm's economic profit at this output?
c. What is the firm's average cost at this output?
d. If other firms in the lamp industry have the same cost function as this firm, is the industry in equilibrium? Why or why not?

Solution:

a. $MC = 20 + 10Q$. Given that the firm is perfectly competitive, then use the

$P = MC$ rule to find the equilibrium quantity. That is, $20 + 10Q = 50$.

Thus $Q = 3$. Moreover, we observe that the MC curve is an increasing function of Q since it is a straight line with a positive slope of 10. Thus, the quantity $Q = 3$ lamps per hour maximizes profit.

b. $\text{Profit} = TR(Q = 3) - TC(Q = 3) = 50(3) – [1{,}000 + 20(3) + 5(3)^2] = –\$955$

c. $ATC = [1{,}000 + 20(3) + 5(3)^2]/3 = \$368.33$

d. None of the firms are covering their fixed costs of $1,000. In fact, the firms are experiencing big losses relative to revenues, so the industry cannot be in equilibrium. Firms will exit until the price increases to the minimum average cost of the remaining firms in the industry (to about $161.42).

2) The long- run supply curve for a particular type of kitchen knife is a horizontal line at a price of $3 per knife. The demand curve for such a kitchen knife is

$Q_D = 50 - 2P$

where Q_D is the quantity of knives demanded (in millions per year) and P is the price per knife (in dollars).

a. What is the equilibrium output of such knives?
b. If a tax of $1 is imposed on each knife, what is the equilibrium output of such knives? (Assume the tax is collected by the government from the suppliers of knives.)
c. After the tax is imposed, you buy such a knife for $3.75. Is this the long run equilibrium price?

Solution:

a. The demand curve can be rewritten as P_D = 25 - 0.5Q_D, and the supply curve is given as P_S = 3 Setting PD = PS and solving for Q_D, we get Q = 44.

b. The price will now have to cover the $1 tax. Setting $P_D = P_S = 1$ and solving for Q_D, we get Q =42.

c. No, the long- run equilibrium price must be $4. If the price initially is $3.75, the existing firms are only receiving $2.75 per knife after they pay the $1 tax and so must be losing money. Exit from the industry will occur until the net price received by the firms increases back to $3.

3) The Coolidge Corporation is the only producer of a particular type of laser.
The demand curve for its product is
Q= 8,300 - 2.1P
and its total cost function is
$TC = 2{,}200 + 480Q + 20Q^2$
where P is price (in dollars), TC is total cost (in dollars), and Q is monthly output.
a. Derive an expression for the firm's marginal revenue curve.
b. To maximize profit, how many lasers should the firm produce and sell per month?
c. If this number were produced and sold, what would be the firm's monthly profit?

Solution:

a. We have TR = PQ. Let's solve for P in terms of Q. We have 2.1P = 8,300 - Q; that is, P = 3,952.381 - 0.4762Q.

$TR = PQ = (3{,}952.381 - 0.4762Q)Q = 3{,}952.381Q - 0.4762Q^2$
MR = dTR/dQ = 3,952.381 - 0.9524Q

b. Set MR = 3,952.381 - 0.9524Q = MC = dTC/dQ = 480 + 40Q
3,472.381= 40.9524Q
Q = 84.8

c. $Profit = [3{,}952.381 - 0.4762(84.8)](84.8) - [2{,}200 + 480(84.8)\ 20(84.8)^2]$
Profit = 331,737.54 - 186,724.80
Profit = $145,012.74

4) The Madison Corporation, a monopolist, receives a report from a consulting firm concluding that the demand function for its product is
Q = 78 - 1.1P + 2.3Y + 0.9A
where Q is the number of units sold, P is the price of its product (in dollars), Y is per capita income (in thousands of dollars), and A is the firm's advertising expenditure (in thousands of dollars). The firm's average variable cost function is
$AVC = 42 - 8Q + 1.5Q^2$

where AVC is average variable cost (in dollars).

a Can we determine the firm’s marginal cost curve?
b. Can we determine the firm’s marginal revenue curve?
c. If per capita income is $4,000 and advertising expenditure is $200,000,
can we determine the price and output where marginal revenue equals marginal cost? If so, what are they?

Solution:
a. Yes! We only need the TVC to calculate marginal cost. Given the AVC,
we have $TVC = 42Q - 8Q^2 + 1.5Q^3$.
$MC = dTC/dQ = dTVC/dQ = 42 - 16Q + 4.5Q^2$

b. Solving for P gives $P = (78 + 2.3Y + 0.9A)/1.1 - Q/1.1$. Hence,
$TR = PQ = Q(78 + 2.3Y + 0.9A)/1.1 - Q^2/1.1$
$MR = dTR/dQ = (78 + 2.3Y + 0.9A)/1.1 - 2\,Q/1.1$
We need information on per capita income and on advertising expenditures
to determine marginal revenue as a function of Q alone.

c. Set MR = MC.
$MR = 242.9 - 1.82Q = 42 - 16Q + 4.5Q^2 = MC$
$4.5Q^2 - 14.18Q - 200.9 = 0$
Solving this using the quadratic formula, we get Q = 8.44 and P = 235.24.