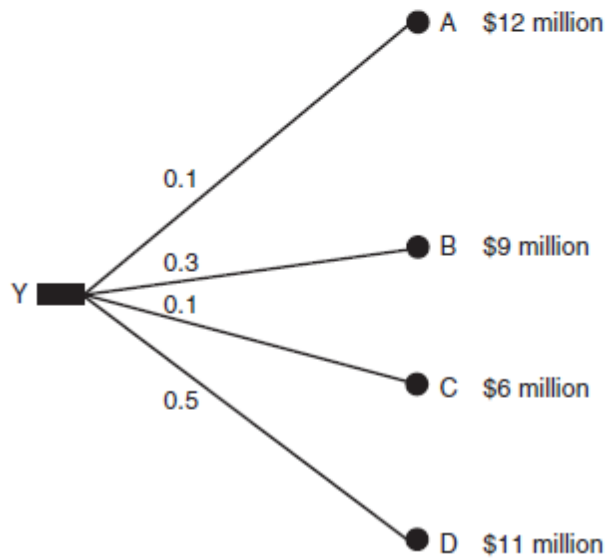
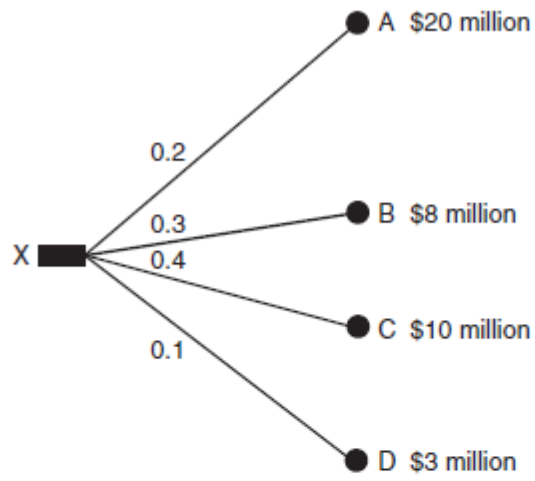


1. The president of the Martin Company is considering two alternative investments,  $X$  and  $Y$ . If each investment is carried out, there are four possible outcomes. The present value of net profit and probability of each outcome follow:

Investment $X$			Investment $Y$		
Outcome	Net Present		Outcome	Net Present	
	Value	Probability		Value	Probability
1	\$20 million	0.2	A	\$12 million	0.1
2	8 million	0.3	B	9 million	0.3
3	10 million	0.4	C	6 million	0.1
4	3 million	0.1	D	11 million	0.5

- a. What are the expected present value, standard deviation, and coefficient of variation of investment  $X$ ?
- b. What are the expected present value, standard deviation, and coefficient of variation of investment  $Y$ ?
- c. Which investment is riskier?
- d. The president of the Martin Company has the utility function  $U = 10 + 4P - 0.2P^2$  where  $U$  is utility and  $P$  is net present value. Which investment should she choose?

SOLUTION:



- a.  $E(\text{Investment } X) = (0.2)(20) + (0.3)(8) + (0.4)(10) + (0.1)(3) = \$10.7$  million

$$\sigma_X = [(20 - 10.7)^2(0.2) + (8 - 10.7)^2(0.3) + (10 - 10.7)^2(0.4) + (3 - 10.7)^2(0.1)]^{1/2}$$

$$\sigma_X = 5.0606.$$

$$V_X = \sigma_X / E(X) = 5.0606 / 10.7 = 0.4729$$

- b.  $E(\text{Investment } Y) = (.1)(12) + (.3)(9) + (.1)(6) + .5(11) = \$10$  million

$$\sigma_Y = [(12 - 10)^2(.1) + (9 - 10)^2(.3) + (16 - 10)^2(.1) + (11 - 10)^2(.5)]^{1/2} = 1.673$$

$$V_Y = \sigma_Y / E(p_Y) = 1.673 / 10 = 0.1673$$

- c.  $V_X > V_Y \rightarrow X$  is riskier.

- d. An easy way to compute the expected utility of  $X$  and  $Y$  is to use the linearity property of the expectation operator. Doing so, we have

$$E(U) = 10 + 5E(P) - 0.2E(P^2)$$

For investment  $X$ , we have  $E(X) = \$10.7$  million, so that  $E(P) = \$10.7$  million.

Therefore, we obtain

$E(P^2) = 400(0.2) + 64(0.3) + 100(0.4) + 9(0.1) = 140.1$  and, therefore, expected utility of  $X$  is given by

$$10 + 4(10.7) - 0.2(140.1) = 24.78$$

Similarly, for investment  $Y$ , we obtain

$E(P^2) = 144(0.1) + 81(0.3) + 36(0.1) + 121(0.5) = 102.8$ . Therefore,  $E[U(X)] = 10 + 40 - 0.2(102.8) = 29.44$ .

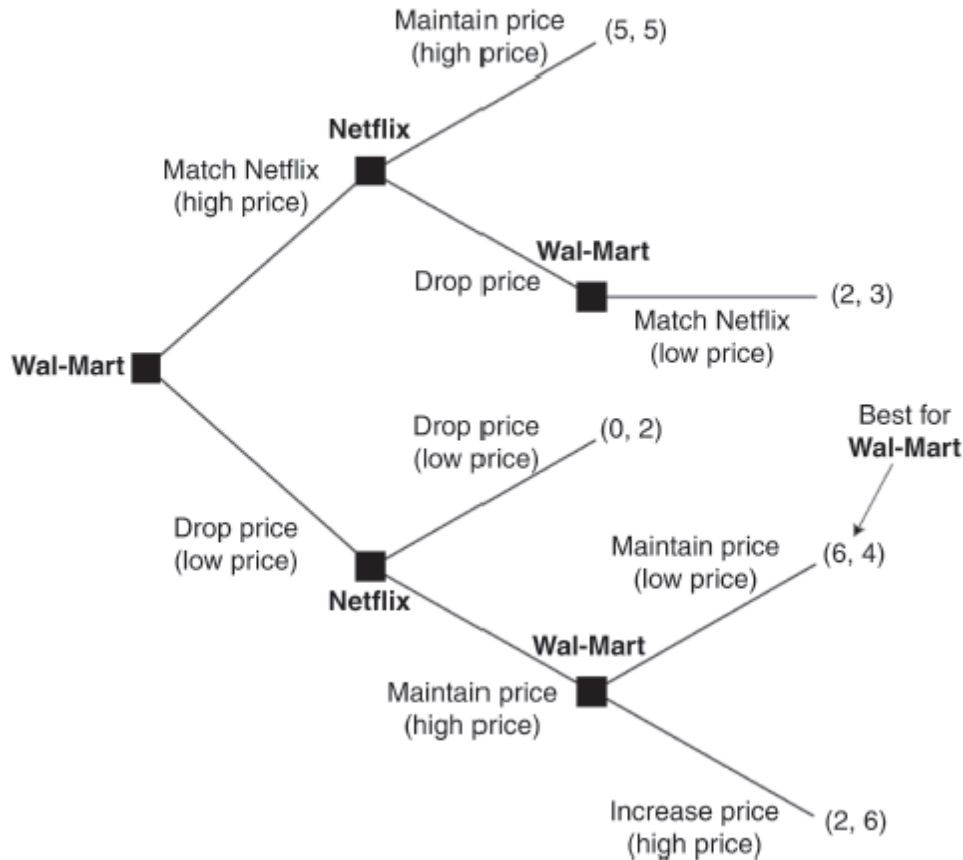
Since investment  $Y$  has a higher expected utility than investment  $X$ , then investment  $Y$  should be chosen assuming the investor is rational and therefore maximizes expected utility.

2) The *New York Times* reports that Wal- Mart has decided to challenge Netflix and enter the online DVD- by- mail market. Because of economies of scale, Wal- Mart has a slight cost advantage relative to Netflix. Wal- Mart is considering the use of a limit pricing strategy. It can enter the market by matching Netflix on price. If it does, and Netflix maintains its price, then both firms would earn \$5 million. But if Netflix drops its price in response, Wal- Mart would have to follow and would earn \$2 million; Netflix would earn \$3 million. Or Wal- Mart could enter the market with a price that is below Netflix's current price but above its marginal cost. If it does, Netflix would make one of two moves. It could reduce its price to below that of Wal- Mart. If it does, Wal- Mart will earn a profit of \$0, and Netflix will earn a profit of \$2 million. Or Netflix could keep its present price. If Netflix keeps its present price, Wal- Mart can keep its present price and earn \$6 million (while Netflix earns \$4 million). Or Wal- Mart can increase its price and earn \$2 million while Netflix earns \$6 million.

- Draw the extensive form of this game and solve it.
- Draw the game's matrix form and identify any Nash equilibria.

**SOLUTION:**

a. Extensive form: Note Wal-Mart payoffs appear first.



a. The game can be solved using backward induction from the extensive form, starting from the last nodes. Wal-Mart should enter and drop its price. Netflix will not drop its price because it would only get 2. If Netflix maintains its price, it will either get 4 or 6, both of which are better than 2. If Netflix maintains its original price (after Wal-Mart has entered with a dropped price), Wal-Mart will maintain that dropped price because it yields them 6 (as opposed to 2 if Wal-Mart increased its price). If Wal-Mart initially matches Netflix's price, Netflix will maintain its initial price and earn 5. If Netflix drops its initial price, Netflix will only earn 3. Thus, Wal-Mart will earn 5. Since Wal-Mart earns 6 by initially lowering its price, it is better for Wal-Mart to drop its price when entering and not match Netflix's price when entering.

b. The solution to this part is obtained in the same way as in the previous exercises. The main point here is to list all the strategy sets for each player. The solution is the same as in part (a) using backward induction.

