OBJECTIVE

- Explain how managers should determine the optimal method of production by applying an understanding of production processes

Theory of the Firm

We said we were going to deal with most problems by thinking about some decision makers who have goals, and who face certain limitations on their abilities to achieve these goals. We try to describe some optimization process by which agents decide what actions to take to achieve those goals subject to those constraints. That process tells us something about behavior. Then we change the limitations that the agents face and ask what changes in behavior will occur.

In the case of consumers, they have preferences and utility functions and they face a budget constraint. We asked what were the changes in behavior due to changes in the budget constraint.

This same methodology can be used to discuss the theory of the firm. **There are different environments in which a firm operates.** We will talk about some of those different paradigms (competitive firms, monopolistic firms, oligopolistic firms). Within some of those environments we will talk about how **a change in the limitations could lead to a change in behavior.** We can ask for example, if a change in the prices the firm pays for its inputs or charges for its outputs may lead to a change in what the firm chooses to do.

Constraints. The constraints of the firm will have something to do with the environment. For example, pricing opportunities, technologies, or reactions of other firms. Changes in Technologies are things that can change the firm constraints.

Goals. Some possible objectives for a firm are cost minimization, profit maximization, maximization of sales, or maximization of revenues. An example of a firm that could have as a goal cost minimization is a firm run by the government, say tren urbano. Tren urbano is not in business to maximize profits, but it might be in business to provide certain level of service in the least expensive way. Or it might be mandated to provide the maximum level of service subject to not making a loss. We have many possible models. Each will be relevant under a particular kind of situation.

Behavior. We will be asking questions such as how much does the firm produce, what price does it charge, and how many inputs does it buy.

Technology

Production functions.

A production function is a function that states the relationship between a set of inputs the firm uses and outputs.

When we think of the case of the consumer, the production function are similar to utility functions. Utility functions allow us to derive utility as function of goods consumed while production functions allow us to derive a level of production from a given level of inputs.

Output=f(inputs)=Q=f(x_1,x_2,……,x_n)

PRODUCTION PROCESSES

- Production processes include all activities associated with providing goods and services, including
  - Employment practices
  - Acquisition of capital resources
  - Product distribution
  - Managing intellectual resources

PRODUCTION PROCESSES

- Production processes define the relationships between resources used and goods and services produced per time period.
  - Managers exert control over production costs by understanding and managing production technology.

PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- A production function shows the maximum amount that can be produced per time period with the best available technology from any given combination of inputs.
  - Table
  - Graph
  - Equation

PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- Production Function Example
  - $Q = f(X_1, X_2)$
    - Q = Output rate
    - X_1 = Input 1 usage rate
    - X_2 = Input 2 usage rate
  - $Q = 30L + 20L^2 – L^3$
    - Q = Hundreds of parts produced per year
    - L = Number of machinists hired
    - Fixed Capital = Five machine tools

TABLE 5.1

Output of Metal Parts When Various Amounts of Labor Are Applied to Five Machine Tools, Thomas Machine Company

Amount of Labor (L)	Amount of Capital (Number of Machines)	Output of Parts (Q, hundreds per year)
0	5	0
1	5	49
2	5	132
3	5	243
4	5	376
5	5	525
6	5	684
6.67	5	792.59
7	5	847
8	5	1,008
9	5	1,161
10	5	1,300
11	5	1,419
12	5	1,512
13	5	1,573
14	5	1,596
15	5	1,575

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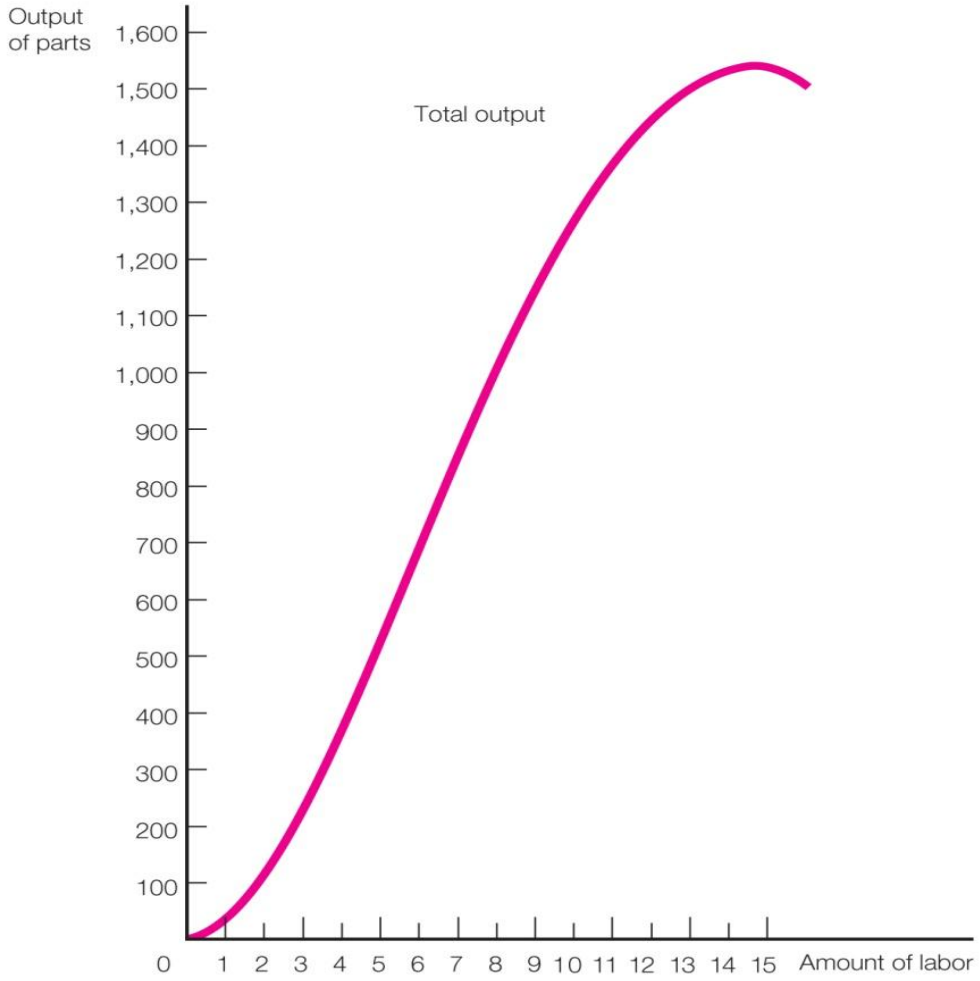
RELATIONSHIP BETWEEN TOTAL OUTPUT AND AMOUNT OF LABOR USED ON FIVE MACHINE TOOLS, THOMAS MACHINE COMPANY

FIGURE 5.1

Relationship between Total Output and Amount of Labor Used on Five Machine Tools, Thomas Machine Company



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PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- Unit Functions
  - Average Product of Labor = $AP_L = Q/L$
    - Common measuring device for estimating the units of output, on average, per worker

PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- Unit Functions (cont'd)
  - Marginal Product of Labor = $MP_L = \Delta Q/\Delta L$
    - Metric for estimating the efficiency of each input in which the input's MP is equal to the incremental change in output created by a small increase in the input
    - Using calculus (assumes that labor can be varied continuously): MP = dQ/dL

PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- Unit Functions (cont'd)
  - Unit function examples from $Q = 30L + 20L^2 – L^3$
    - $AP_L = 30 + 20L – L^2$
    - Using calculus: $MP_L = 30 + 40L – 3L^2$
    - AP_L is at a maximum, and $MP_L = AP_L$, at $L = 10$ and $MP_L = AP_L = 130$
    - MP_L is at a maximum at $L = 6.67$ and $MP_L = 163.33$

PRODUCTION FUNCTION WITH ONE VARIABLE INPUT

- Unit Functions (cont'd)
  - Why does $MP_L = AP_L$ when AP_L is at a maximum?
    - If $MP_L > AP_L$, then AP_L must be increasing
    - If $MP_L < AP_L$, then AP_L must be decreasing

TABLE 5.2

Average and Marginal Products of Labor, Thomas Machine Company

Amount of Labor (Units)	Amount of Capital (Number of Machines)	Output of Parts (Q, Hundreds of Parts)	Average Product (Q/L)	Marginal Product ($\Delta Q/\Delta L$)[a]	Marginal Product (dQ/dL)[a]
0	5	0	—	—	—
1	5	49	49	49	67
2	5	132	66	83	98
3	5	243	81	111	123
4	5	376	94	133	142
5	5	525	105	149	155
6	5	684	114	159	162
6.67	5	792.59	118.89	162.89	163.33
7	5	847	121	163	163
8	5	1,008	126	161	158
9	5	1,161	129	153	147
10	5	1,300	130	139	130
11	5	1,419	129	119	107
12	5	1,512	126	93	78
13	5	1,573	121	61	43
14	5	1,596	114	23	2
15	5	1,575	105	−19	−45

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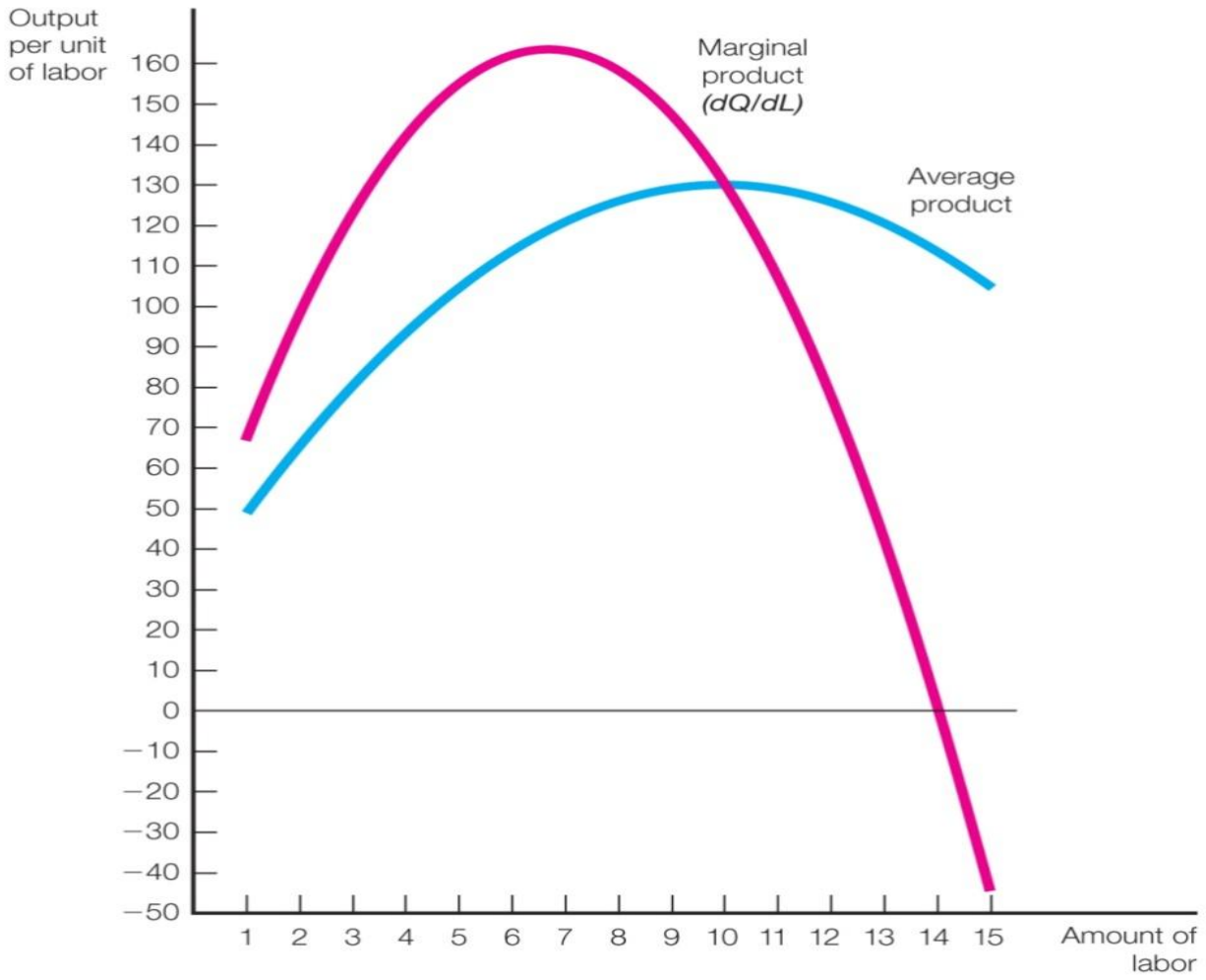
AVERAGE AND MARGINAL PRODUCT CURVES FOR LABOR

FIGURE 5.2

Average and Marginal Product Curves for Labor



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THE PRODUCTIONN FUNCTION WITH TWO VARIABLE INPUTS

- $Q = f(X_1, X_2)$
  - Q = Output rate
  - X_1 = Input 1 usage rate
  - X_2 = Input 2 usage rate
  - $AP_1 = Q/X_1$ and $MP_1 = \Delta Q/\Delta X_1$ or dQ/dX_1
  - $AP_2 = Q/X_2$ and $MP_2 = \Delta Q/\Delta X_2$ or dQ/dX_2

Below we have an example of a two variable Cobb-Douglas production function.

$q = f(K,L)=K^{\alpha}L^{\beta}$

Where K is capital and L is labor. Like utility we can learn a lot by analyzing these functions on the margin.

Marginal prodauct

Marginal product is the added production gained from increasing one input. The concept is similar to Marginal Utility.

$$MP_K = \frac{\partial q}{\partial k} = f_K$$

$$MP_L = \frac{\partial q}{\partial l} = f_l$$

THE LAW OF DIMINISHING MARGINAL RETURNS

- Law of diminishing returns
  - When managers add equal increments of an input while holding other input levels constant, the incremental gains to output eventually get smaller.

Diminishing Marginal Product

For the most part we will assume diminishing marginal product in much the same way we assumed diminishing marginal utility. What this means is as we add more inputs production may go up but the last unit of input adds less to production than the unit of input before. What this means is that the first derivative may be positive but the second should be negative.

$$\frac{\partial MP_K}{\partial K} = \frac{\partial^2 q}{\partial k^2} = f_{kk} \langle 0$$

$$\frac{\partial MP_L}{\partial L} = \frac{\partial^2 q}{\partial L^2} = f_{LL} \langle 0$$

As we have seen Average product AP is

$$AP_L = \frac{q}{L} = \frac{f(k, L)}{L}$$

$$AP_K = \frac{q}{K} = \frac{f(K, L)}{K}$$

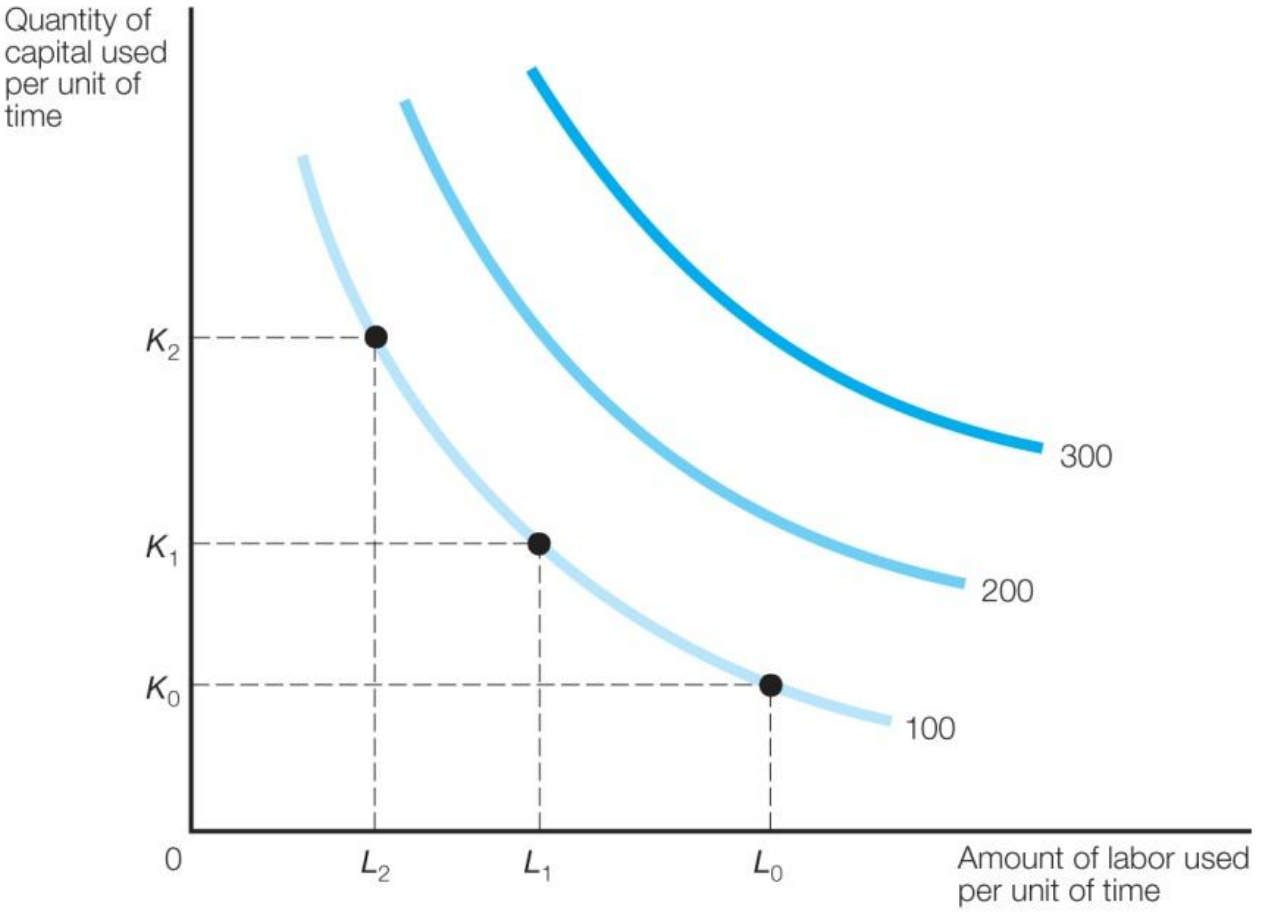
ISOQUANTS

- Isoquant: Curve showing all possible (efficient) input bundles capable of producing a given output level
  - Graphically constructed by cutting horizontally through the production surface at a given output level

ISOQUANTS

FIGURE 5.4

Isoquants



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ISOQUANTS

- Properties
  - Isoquants farther from the origin represent higher output levels.
  - Given a continuous production function, every possible input bundle is on an isoquant and there is an infinite number of possible input combinations.
  - Isoquants slope downward to the left and are convex to the origin.

Isoquants

Say we have a single output good and two inputs y_1 and y_2. We may want to look at combinations of those two inputs that can produce the same level of output. Say the points in figure A below are two ways we can produce the same amount of omelets. Each one uses different proportions of eggs and labor. One uses a relatively smaller amount of eggs and more labor than the other one.

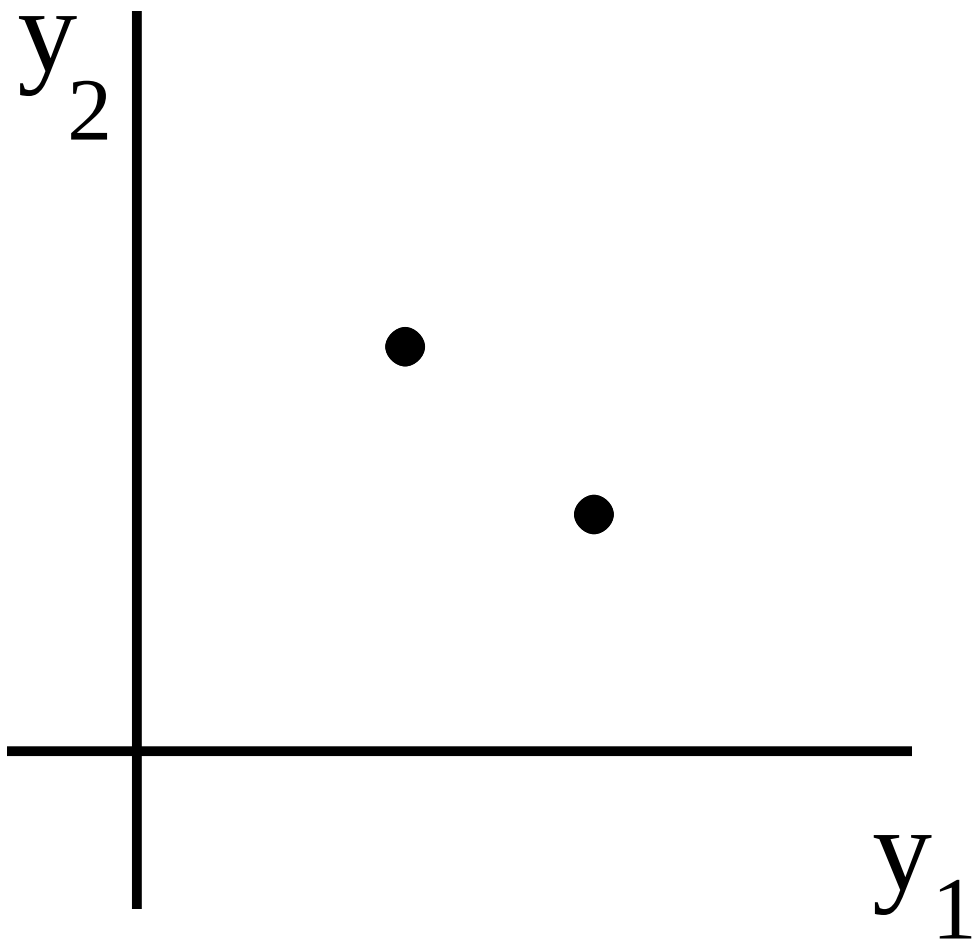
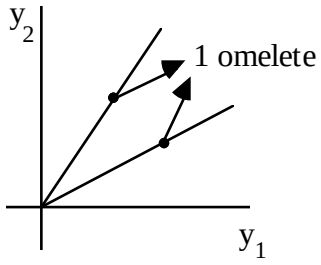
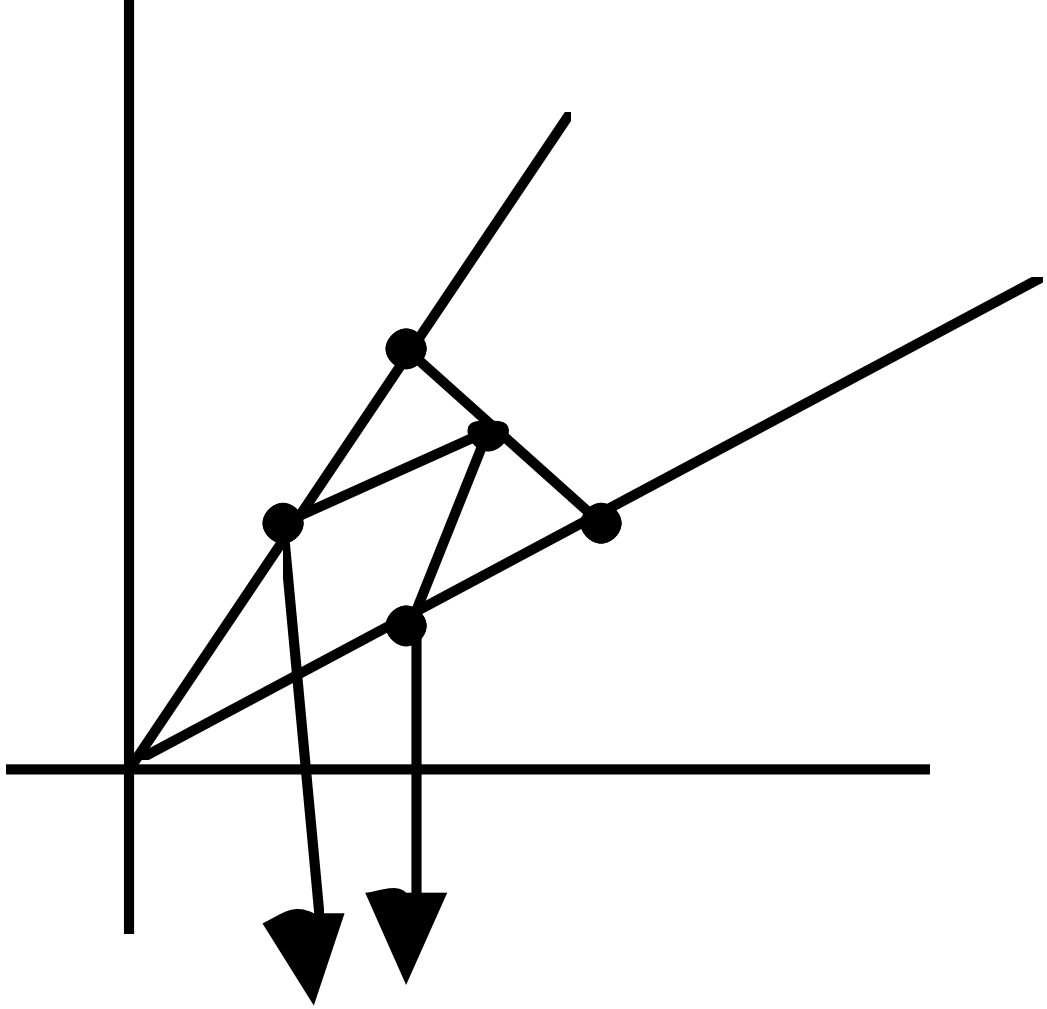


Figure A

I may want to assume constant returns to scale. This would mean that I can produce anywhere along the rays from the origin in figure below. So far, all my technology has described is a production set that consists of these 2 rays. (The arrows in Figure B are just referring to the fact that both points produce 1 omelet.)


y2
1 omelete
y1

We may also want to assume convexity. This would mean that I can also produce one omelet using a linear combination of these two techniques. For example, I can produce half omelet using one technique and another half omelet using the other technique. If we have additivity, we can produce a whole omelet by producing half an omelet in each particular way (see figure C). Or I can produce a whole omelet using a convex combination of these two techniques.



1/2 omelete

Figure C

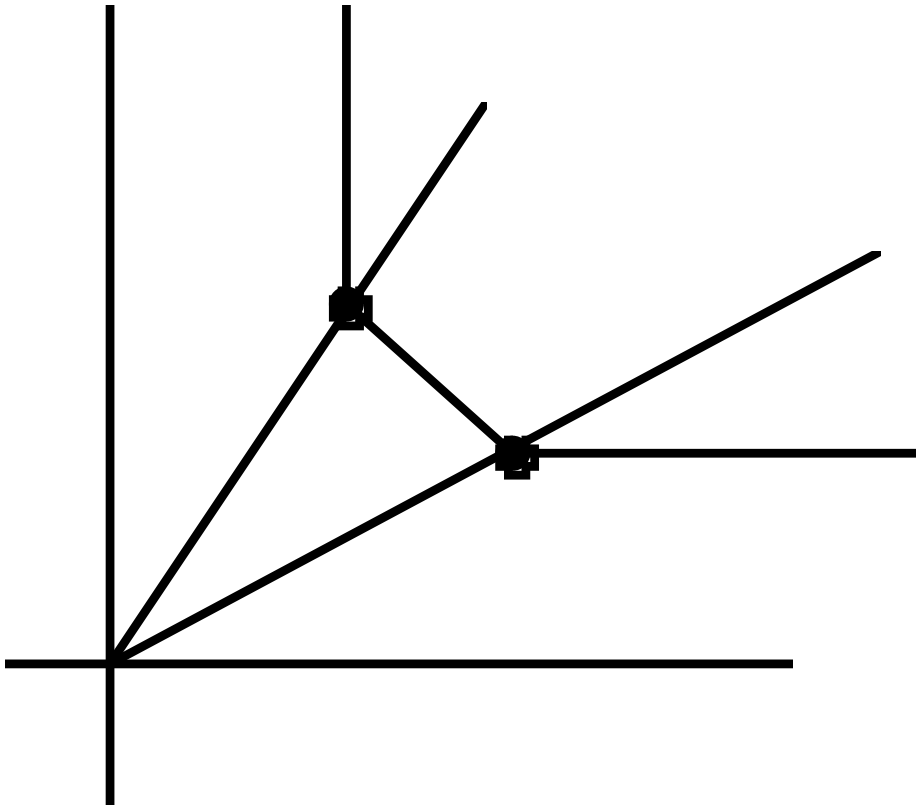


Figure D

If I add free disposal it means that I can also produce one omelet using arbitrarily more of factor 1 and arbitrarily more of factor 2 (see figure D).

If I had three different ways of producing omelets by taking convex combinations we would get figure E.

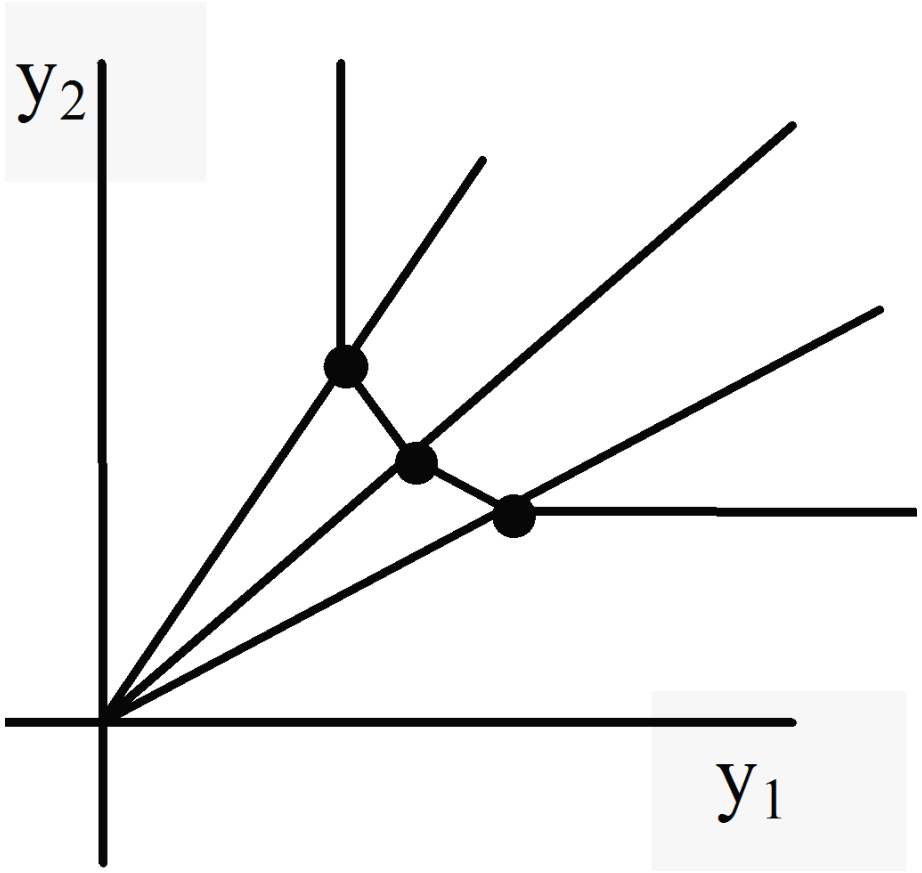


Figure E

I am going to produce a convex hull from all the different activities that are available to me. If I have several ways of producing one omelet, **the lower hull would be all the efficient ways of producing a single omelet** (see figure F). In general, if I have lots of different activities, ultimately I would end up with some convex surface like the one in figure F, and it would be called the **unit isoquant**.

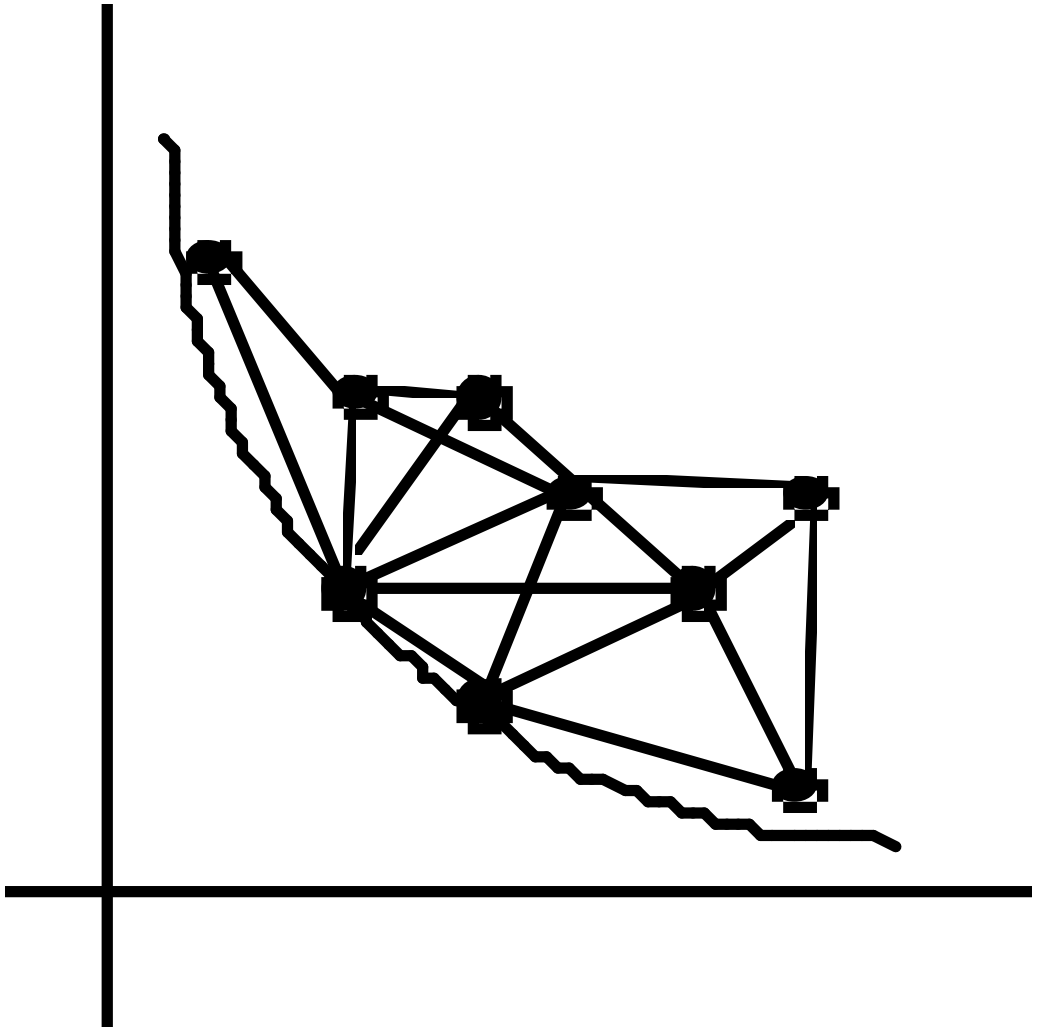
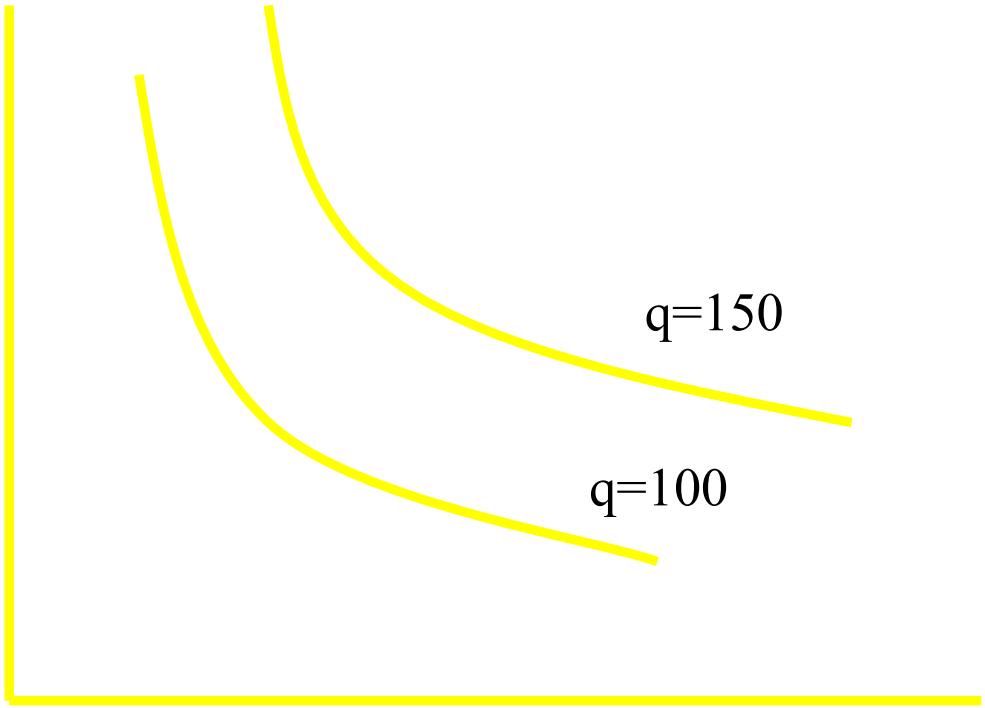


Figure F

In general an isoquant depics a situation where the same level of production can be achieved using different combinations of inputs.



Notice this isoquant looks a lot like an indifference curve. The difference is here we are analyzing production while we used the indifference curve to analyze utility. Of course the indifference curve is derived from the utility function while the isoquant is derived from the production function.

MARGINAL RATE OF TECHNICAL SUBSTITUTION

- Marginal rate of technical substitution (MRTS): Shows the rate at which one input is substituted for another (with output remaining constant)
- $Q = f(X_1, X_2)$
  - MRTS = $-\Delta X_2/\Delta X_1$ with Q held constant and X_2 on the vertical axis
  - MRTS = MP_1/MP_2
  - MRTS = Absolute value of the slope of an isoquant

MARGINAL RATE OF TECHNICAL SUBSTITUTION

- MRTS and isoquants (with X_2 on the vertical axis)
  - If the MRTS is large, it takes a lot of X_2 to substitute for one unit of X_1, and isoquants will be steep.
  - If the MRTS is small, it takes little X_2 to substitute for one unit of X_1, and isoquants will be flat.

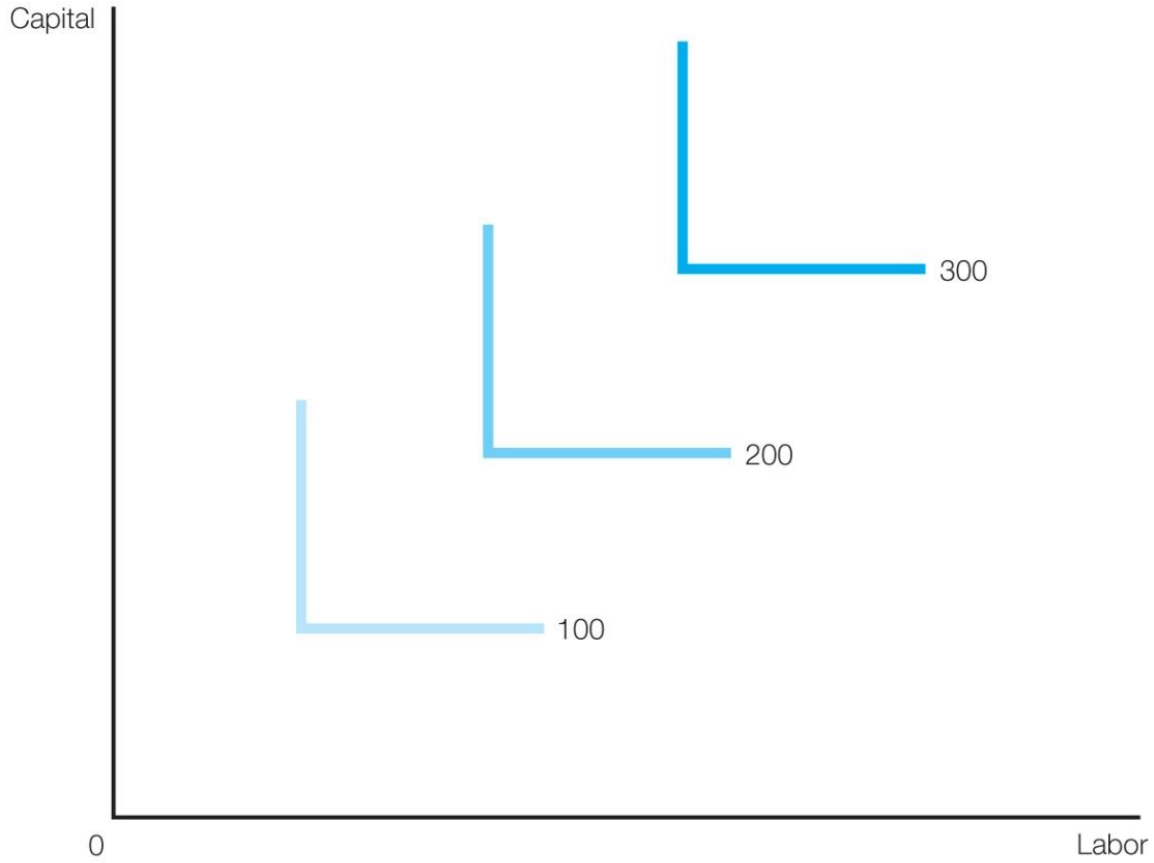
MARGINAL RATE OF TECHNICAL SUBSTITUTION

- MRTS and isoquants (with X_2 on the vertical axis) (cont'd)
  - If X_1 and X_2 are perfect substitutes, MRTS is constant, and isoquants will be straight lines.
  - If X_1 and X_2 are perfect complements, no substitution is possible, MRTS is undefined, and isoquants will be right angles.

ISOQUANTS IN THE CASE OF FIXED PROPORTIONS

FIGURE 5.5

Isoquants in the Case of Fixed Proportions



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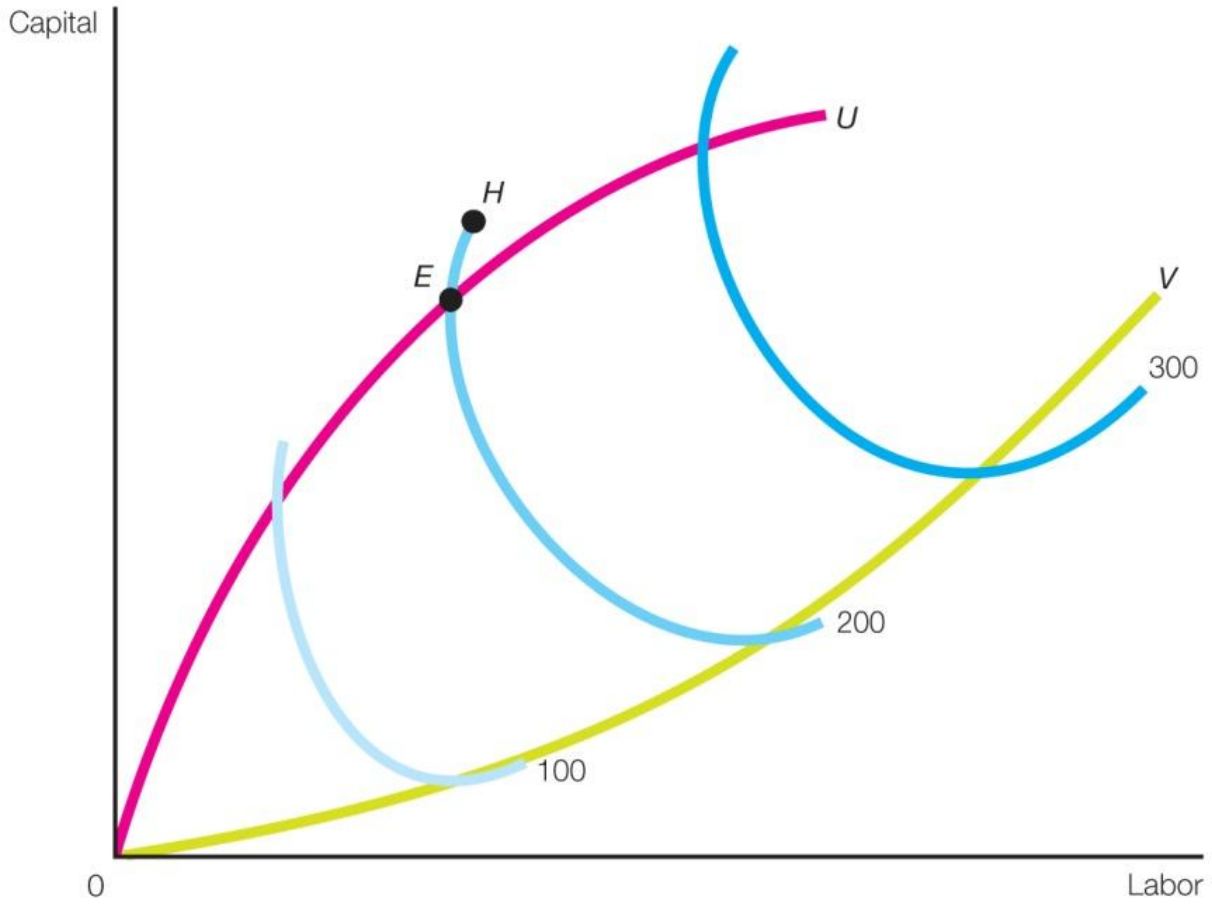
MARGINAL RATE OF TECHNICAL SUBSTITUTION

- Ridge Lines
  - Ridge lines: the lines that profit-maximizing firms operate within because outside them, marginal products of inputs are negative
  - Economic region of production is located within the ridge lines.

ECONOMIC REGION OF PRODUCTION

FIGURE 5.6

Economic Region of Production



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Marginal Rate of Technical Substitution (MRTS)

The MRTS is simply the rate at which the firm has to substitute one input for another while keeping production constant. In other words it is the slope of the isoquant. Again it is a very similar concept to the marginal rate of substitution in utility theory.

$$MRTS = \frac{-dK}{dL}\Big|_{q=q_0}$$

To derive it take the total differential of the production function. Notice we end up with the marginal product of labor and capital multiplied by the change in labor and capital respectively.

$$dq = \frac{\partial f}{\partial L} dL + \frac{\partial f}{\partial K} dK = MP_L dL + MP_K dK$$

Since q is constant $q = q_0$

$dq_0 = 0$

Therefore

$MP_L dL = - MP_K dK$

$$\Rightarrow \quad -\frac{dK}{dL}\bigg|_{q=q_0} MRTS = \frac{MP_L}{MP_K}$$

RETURNS TO SCALE

This concept allows us to exam what happens to production when all inputs are increase by the same percentage.

$f(tK, tL) = tf(K, L)$ Constant

$f(tK, tL) < tf(K, L)$ Decreasing

$f(tK, tL) > tf(K, L)$ Increasing