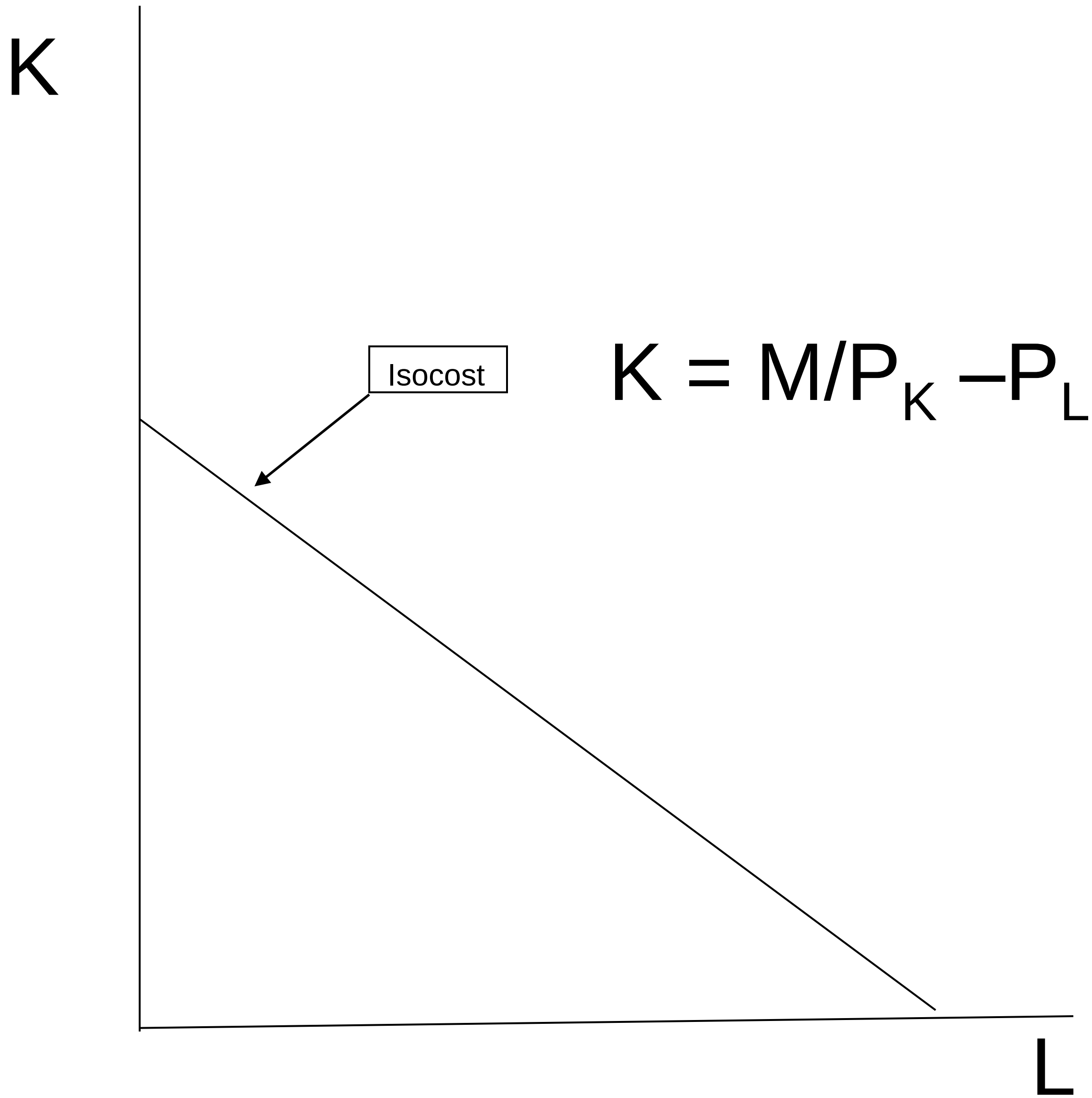
THE OPTIMAL COMBINATION OF INPUTS

- Isocost curve: Curve showing all the input bundles that can be purchased at a specified cost
  - $P_L L + P_K K = M$
    - L = Labor use rate
    - P_L = Price of labor
    - K = Capital use rate
    - P_K = Price of capital
    - M = Total outlay

THE OPTIMAL COMBINATION OF INPUTS

- Isocost curve (cont'd)
  - $K = M/P_K – (P_L/P_K)L$
    - Vertical intercept = M/P_K
    - Horizontal intercept = M/P_L
    - Slope = $– P_L/P_K$

Graphically:

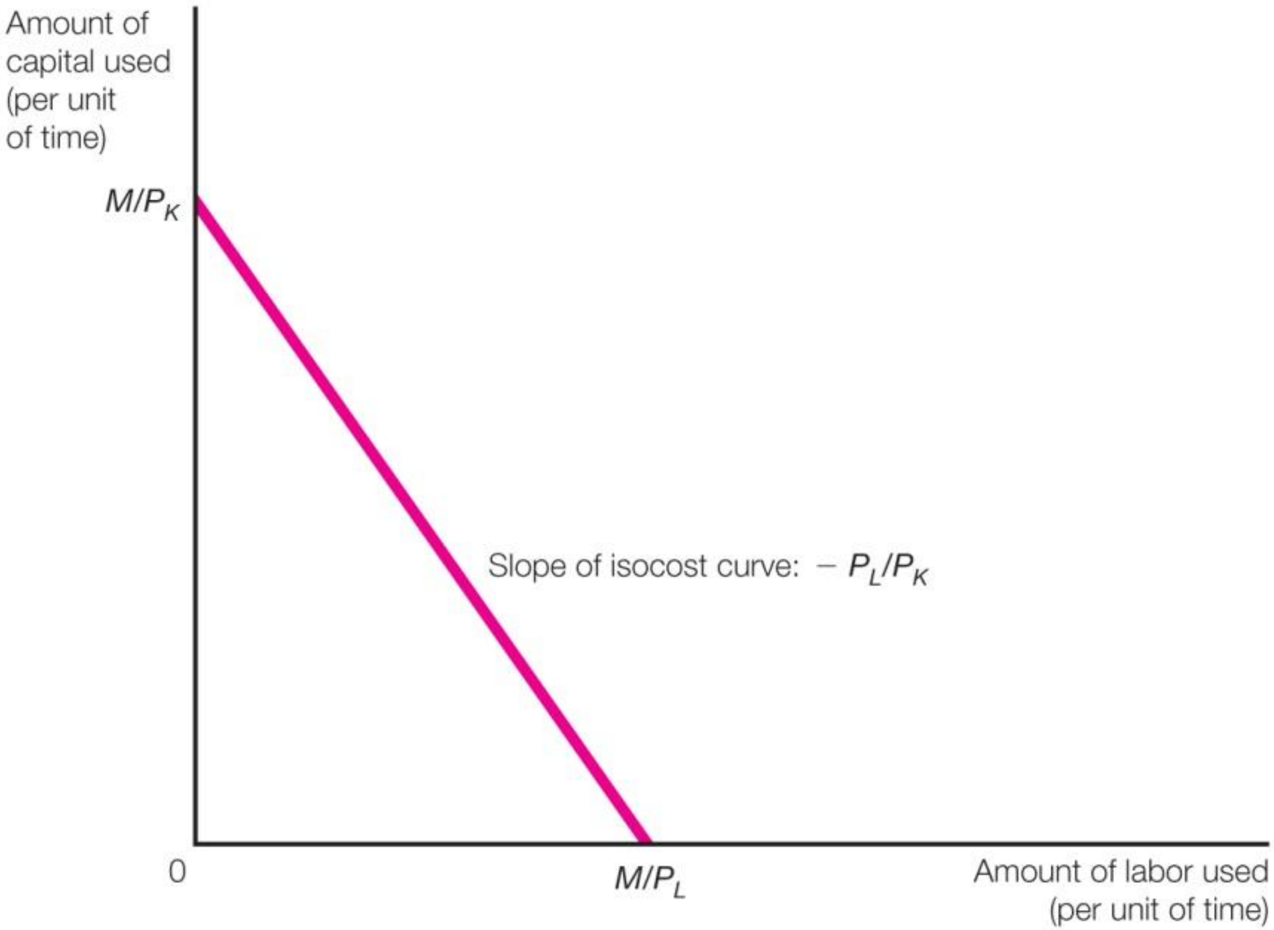


$$K = M/P_K - P_L L/P_K$$

ISOCOST CURVE

FIGURE 5.7

Isocost Curve



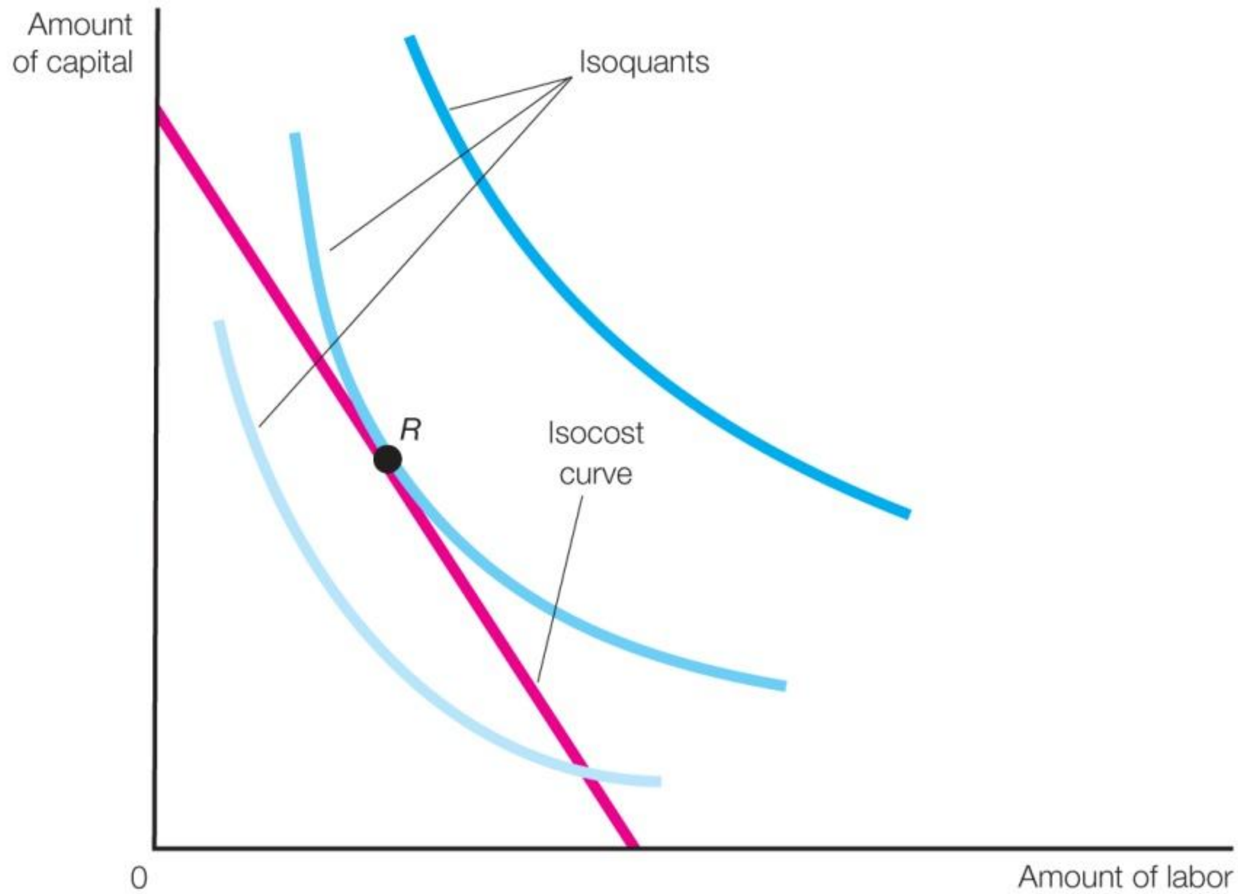
THE OPTIMAL COMBINATION OF INPUTS

- Optimal Combination of Inputs
  - Tangency between isocost and isoquant
    - $MRTS = MP_L/MP_K = P_L/P_K$
    - $MP_L/P_L = MP_K/P_K$
    - Marginal product per dollar spent should be the same for all inputs.
    - $MP_a/P_a = MP_b/P_b = \cdots = MP_n/P_n$

MAXIMIZATION OF OUTPUT FOR A GIVEN COST

FIGURE 5.8

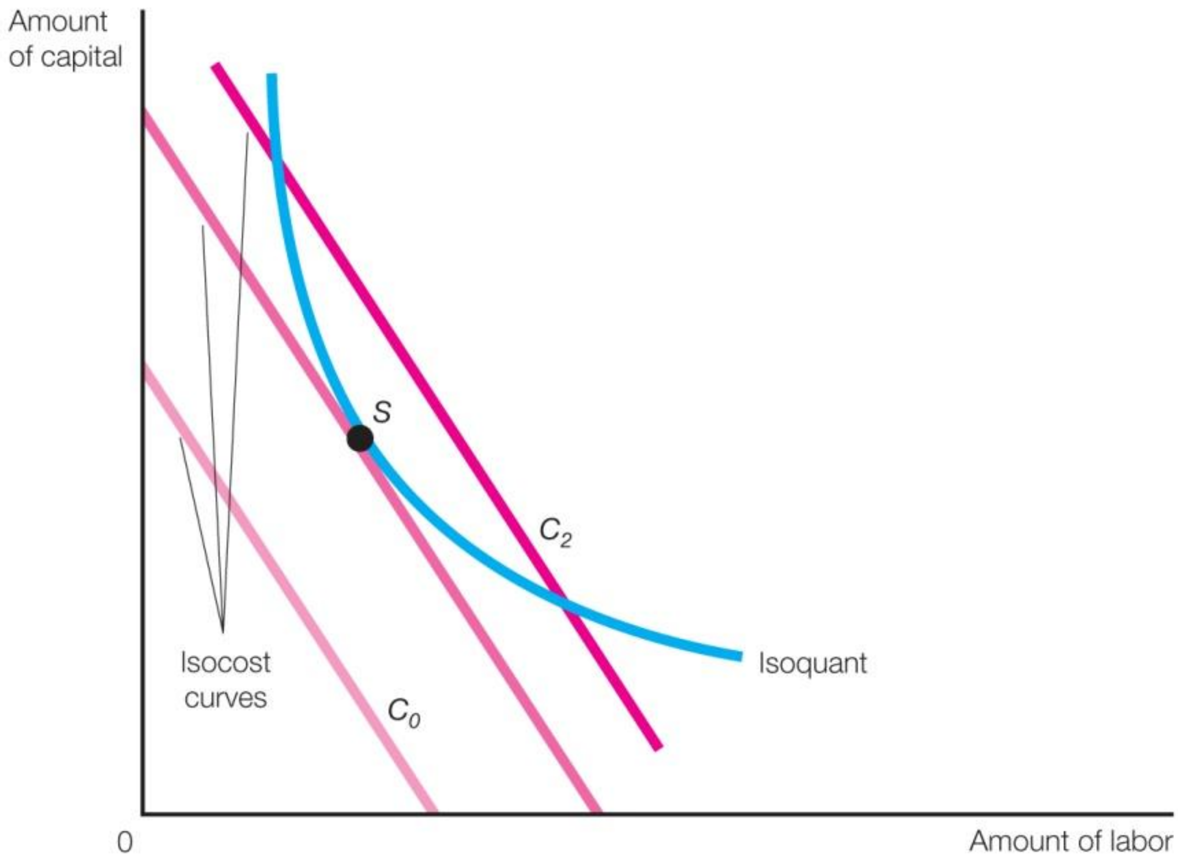
Maximization of Output for a Given Cost



MINIMIZATION OF COST FOR A GIVEN OUTPUT

FIGURE 5.9

Minimization of Cost for a Given Output



Cost minimization occurs at the point of tangency.

Expansion path:

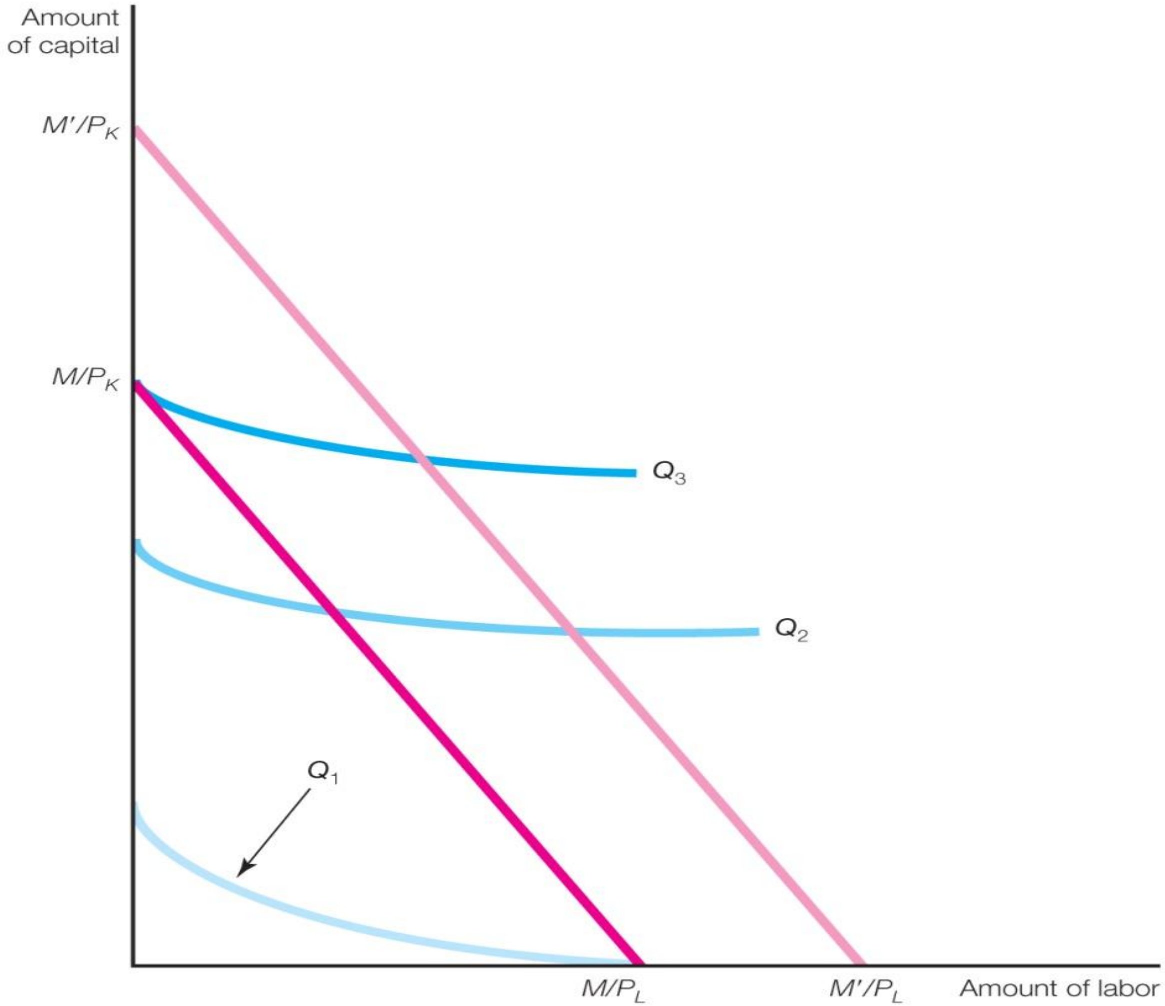
CORNER SOLUTIONS

- Optimal input combination does not occur at a point of tangency between isocost and isoquant curves.
  - In a two-input case, one of the inputs will not be used at all in production.

A CORNER SOLUTION WHERE ONLY ONE INPUT IS USED

FIGURE 5.10

A Corner Solution Where Only One Input Is Used



RETURNS TO SCALE

- Long-run effect of an equal proportional increase in all inputs
  - Increasing returns to scale: When output increases by a larger proportion than inputs
  - Decreasing returns to scale: When output increases by a smaller proportion than inputs
  - Constant returns to scale: When output increases by the same proportion as inputs

RETURNS TO SCALE

- Sources of increasing returns to scale
  - Indivisibilities: Some technologies can only be implemented at a large scale of production.
  - Subdivision of tasks: Larger scale allows increased division of tasks and increases specialization.

RETURNS TO SCALE

- Sources of increasing returns to scale (cont’d)
  - Probabilistic efficiencies: Law of large numbers may reduce risk as scale increases.
  - Geometric relationships: Doubling the size of a box from 1 x 1 x 1 to 2 x 2 x 2 multiplies the surface area by four times (from 3 to 12) but increases the volume by eight times (from 1 to 8). This applies to storage devices, transportation devices, etc.

RETURNS TO SCALE

- Sources of decreasing returns to scale
  - Coordination inefficiencies: Larger organizations are more difficult to manage.
  - Incentive problems: Designing efficient compensation systems in large organizations is difficult.

THE OUTPUT ELASTICITY

- Output elasticity: the percentage change in output resulting from a 1 percent increase in all inputs
  - Note: A more common definition of output elasticity is the percentage change in output resulting from a 1 percent increase in a **single** input. Accordingly, the coefficients 0.3 and 0.8 in the Cobb-Douglas function below would be referred to as the output elasticities of labor and capital, respectively.

THE OUTPUT ELASTICITY

- Cobb-Douglas production function example: $Q = 0.8L^{0.3}K^{0.8}$
  - Q = Parts produced by the Lone Star Company per year
  - L = Number of workers
  - K = Amount of capital
  - Output elasticity = 1.1 for infinitesimal changes in inputs
  - Example calculation for 1 percent increase in both inputs
    - $Q' = 0.8(1.01L)^{0.3}(1.01K)^{0.8} = 1.011005484Q$

ESTIMATIONS OF PRODUCTION FUNCTIONS

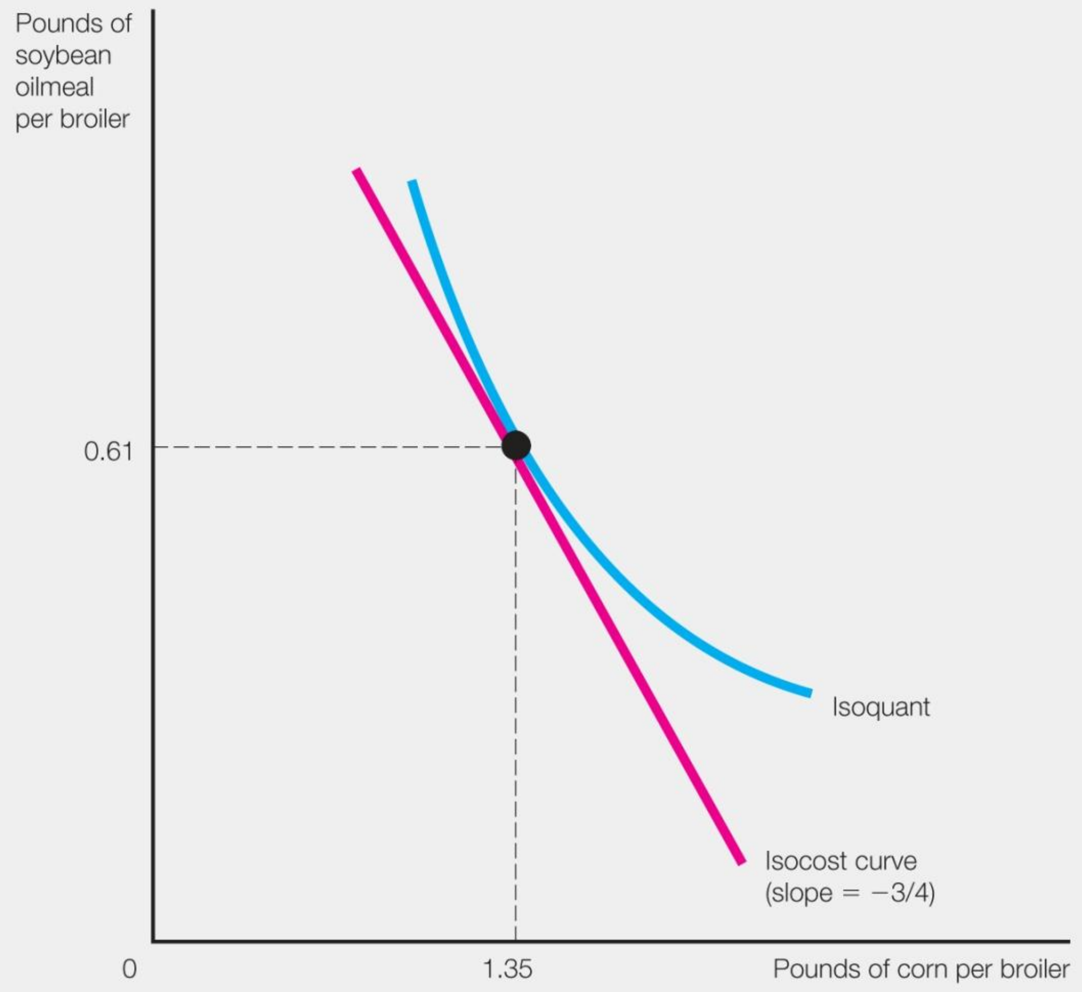
- Cobb-Douglas Mathematical form:

  $Q = aL^{b}K^{c}$

  - $MP_L = \Delta Q/\Delta L = b(Q/L) = b(AP_L)$
  - Linear estimation: $\log Q = \log a + b \log L + c \log K$
  - Returns to scale
    - $b + c > 1$ => increasing returns
    - $b + c = 1$ => constant returns
    - $b + c < 1$ => decreasing returns

ISOQUANT FOR A ONE-POUND WEIGHT GAIN FOR A BROILER AND ISOCOST CURVE IF CORN PRICE IS ¾ OF SOYBEAN OILMEAL PRICE

Isoquant for a One-Pound Weight Gain for a Broiler and Isocost Curve If Corn Price Is ¾ of Soybean Oilmeal Price



OBJECTIVES

- Explain how managers can use their knowledge of the relationship between costs and output to make decisions that maximize the value of the firm
- Understand the difference between short-run and long-run costs and the importance of this distinction for managerial decisions

OPPORTUNITY COSTS

- Definitions
  - Opportunity cost doctrine: the inputs' values (when used in their most productive way) together with production costs (the accounting costs of producing a product) determine the economic cost of production
  - Historical cost: the money that managers actually paid for an input

OPPORTUNITY COSTS

- Definitions (cont'd)
  - Explicit costs: the ordinary items accountants include as the firm's expenses
  - Implicit costs: the foregone value of resources that managers did not put to their best use
  - Doctrine of sunk costs: Resources that are spent and cannot be recovered

Historical Cost
Actual cash outlay when the product was purchased.
Use mainly for tax purposes and SEC filings.

Current Costs
Amount that would be paid under prevailing market conditions.

Replacement cost
The amount it takes to replace productive capability using current technology.

Accounting Costs
The simple dollar cost of and input.

Opportunity Cost
Foregone value associated with current rather that next-best use of an asset.

Economic Costs
Accounting cost plus opportunity costs

Explicit cost
Out-of-pocket expenditures

Implicit cost
Non-cash costs.

Incremental Cost
Change in cost caused by a given managerial decision.

Profit contribution
Profit before fixed charges. (How much does a item add to short term operating profits.)

Sunk costs
Cost that does not vary across decision alternatives. These costs should not play a role in determining the optimal course of action for a company.

Cost Function

The cost-output relation.

Short-run cost function

Basis for day-to-day operating decisions.

Short run

Operating period during which at least one input is fixed.

Fixed cost
Expense that does not vary with output.

Variable cost
Expense the fluctuates with output.

Short run cost curves
Cost-output relation for a specific plant and operating environment.

SHORT-RUN COST FUNCTIONS

- Definitions
  - Cost function: Function showing various relationships between input costs and output rate
  - Short run: Where the quantity of at least one input is fixed
  - Long run: Where the quantities of all inputs are variable
  - Fixed inputs: When the quantities of plant and equipment cannot be altered

SHORT-RUN COST FUNCTIONS

- Definitions (cont'd)
  - Scale of plant: This scale is determined by fixed inputs.
  - Variable inputs: Inputs that a manager can vary in quantity in the short run
  - Total fixed cost (TFC): the total cost per period of time incurred for fixed inputs
  - Total variable cost (TVC): the total cost incurred by managers for variable inputs
  - Total cost (TC = TFC + TVC): the sum of total fixed and total variable costs

TABLE 6.1

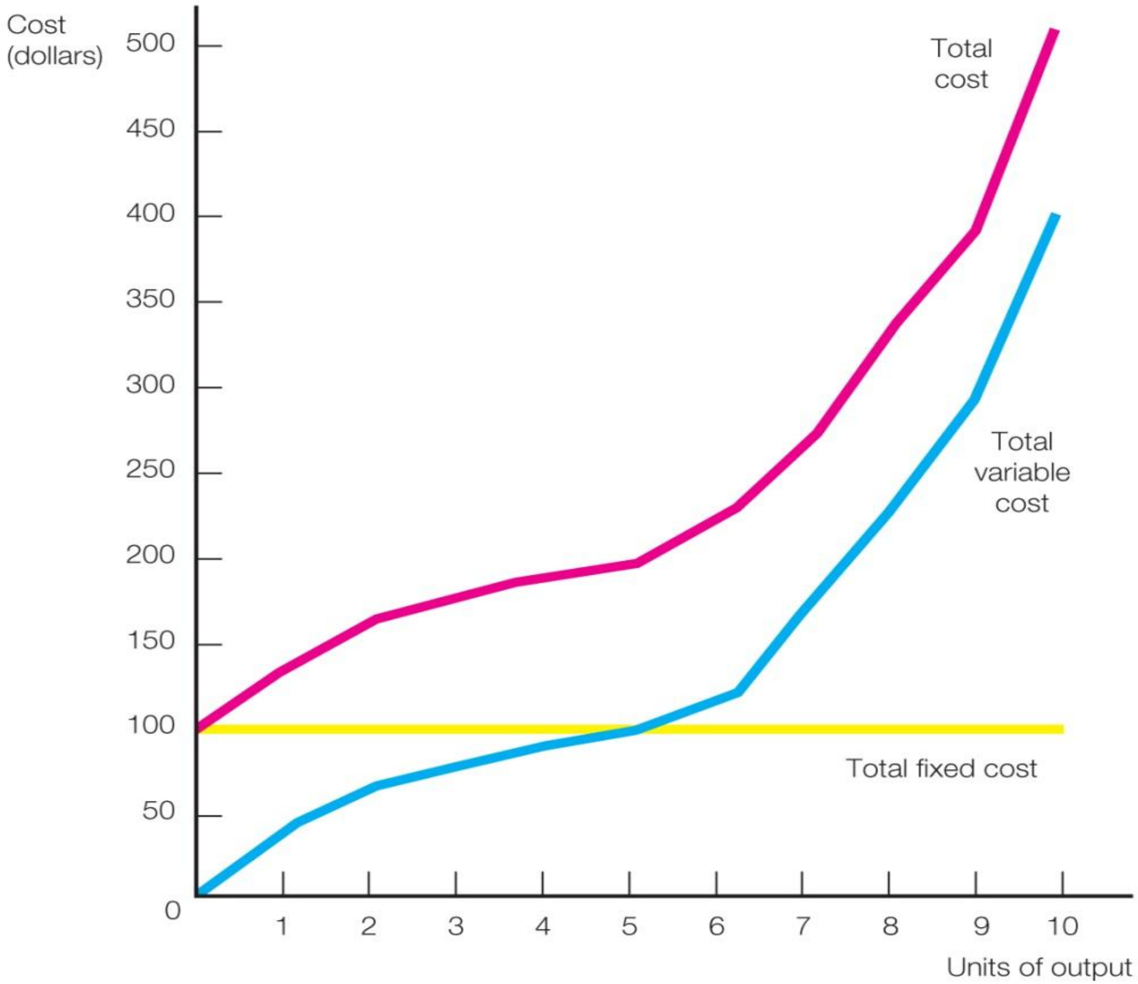
Fixed, Variable, and Total Costs: Media Corporation

Units of Output *Q*	Total Fixed Cost (Dollars per Day) *TFC*	Total Variable Cost (Dollars per Day) *TVC*	Total Cost (Dollars per Day) *TC*
0	100	0	100
1	100	40	140
2	100	64	164
3	100	78	178
4	100	88	188
5	100	100	200
5.5	100	108.625	208.625
6	100	120	220
6.64	100	139.6	239.6
7	100	154	254
8	100	208	308
9	100	288	388
10	100	400	500

FIXED, VARIABLE, AND TOTAL COSTS: MEDIA CORPORATION

FIGURE 6.1

Fixed, Variable, and Total Costs: Media Corporation



Total Cost = Total Fixed cost + Total Variable Costs

TC=TFC + TVC

Average Fixed costs. = AFC =(TFC/ Q)

Average Variable Costs = AVC = TVC/Q

Average Total Cost = ATC = TC/Q

AVERAGE AND MARGINAL COSTS

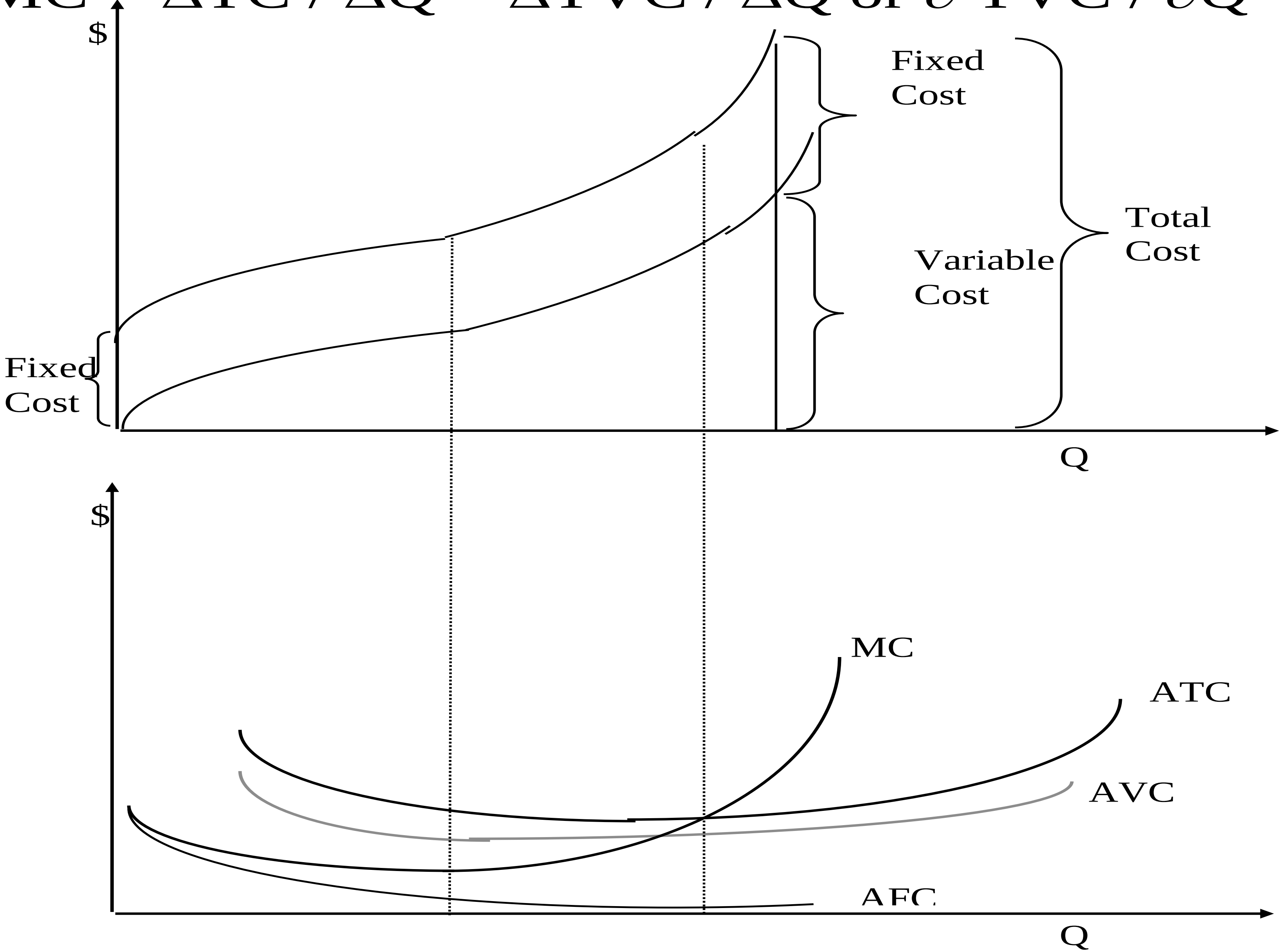
- Definitions
  - Average fixed cost (AFC = TFC/Q): the total fixed cost divided by output
  - Average variable cost (AVC = TVC/Q): the total variable cost divided by output
  - Average total cost (ATC = TC/Q): the total cost divided by output
  - Marginal cost ($MC = \Delta TC/\Delta Q$): the incremental cost of producing an additional unit of output
    - Note: Marginal cost is defined as $\Delta TC/\Delta Q$ for discrete changes in Q and as forgone for continuous changes
      in Q.

AVERAGE AND MARGINAL COSTS

- Relationships
  - Let U be the number of input units used.
  - Let W be the cost per unit of input.
  - AVC = TVC/Q = (WU)/Q =W(U/Q) = W(1/AP) where AP is the average product of U
  - MC = ΔTVC/ΔQ = W(ΔU/ΔQ) = W(1/MP) where MP is the marginal product of U
  - MC = AVC when AVC is at a minimum
  - MC = ATC when ATC is at a minimum

Marginal Cost = MC = $\Delta TC / \Delta Q$ or $\partial TC / \partial Q$

$\Rightarrow MC = \Delta TC / \Delta Q = \Delta TVC / \Delta Q$ or $\partial TVC / \partial Q$



AVERAGE AND MARGINAL COSTS

- Example
  - $AFC = 100/Q$
  - $AVC = 50 - 11Q + Q^2$
  - $ATC = AFC + AVC = 100/Q + 50 - 11Q + Q^2$
  - $MC = \Delta TC/\Delta Q = \Delta TVC/\Delta Q = 50 - 22Q + 3Q^2$
  - AVC is at a minimum where $\Delta AVC/\Delta Q = -11 + 2Q = 0$;
    $MC = AVC = 19.75$ and $Q = 5.5$
  - ATC is at a minimum where
    $$\Delta ATC/\Delta Q = -(100/Q^2) - 11 + 2Q = 0;$$
  - $MC = ATC = 36.11$ and $Q = 6.6$

TABLE 6.2

Average and Marginal Costs: Media Corporation

Units of Output Q	Average Fixed Cost (Dollars) TFC/Q	Average Variable Cost (Dollars) TVC/Q	Average Total Cost (Dollars) TC/Q	Marginal Cost (Dollars) $\Delta TC/\Delta Q$[a]	Marginal Cost (Dollars) dTC/dQ[a]
0	—	—	—	—	—
1	100	40	140	40	31
2	50	32	82	24	18
3	33.33	26	59.33	14	11
4	25	22	47	10	10
5	20	20	40	12	15
5.5	18.18	19.75	37.93		19.75
6	16.67	20	36.67	20	26
6.64	15.06	21.04	36.11		36.11
7	14.29	22	36.29	34	43
8	12.5	26	38.5	54	66
9	11.11	32	43.11	80	95
10	10	40	50	112	130

AVERAGE AND MARGINAL COST CURVES: MEDIA CORPORATION

FIGURE 6.2

Average and Marginal Cost Curves: Media Corporation

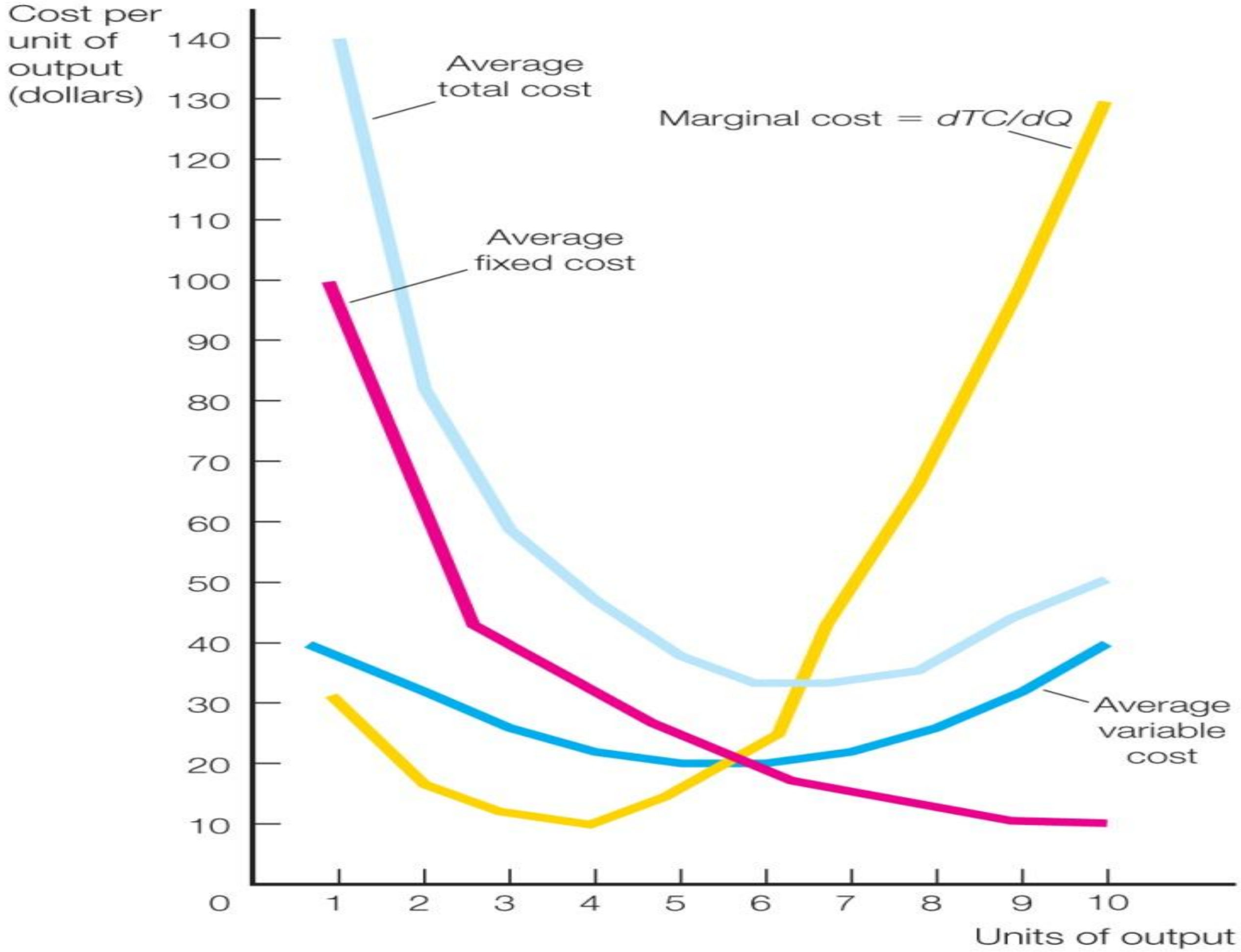


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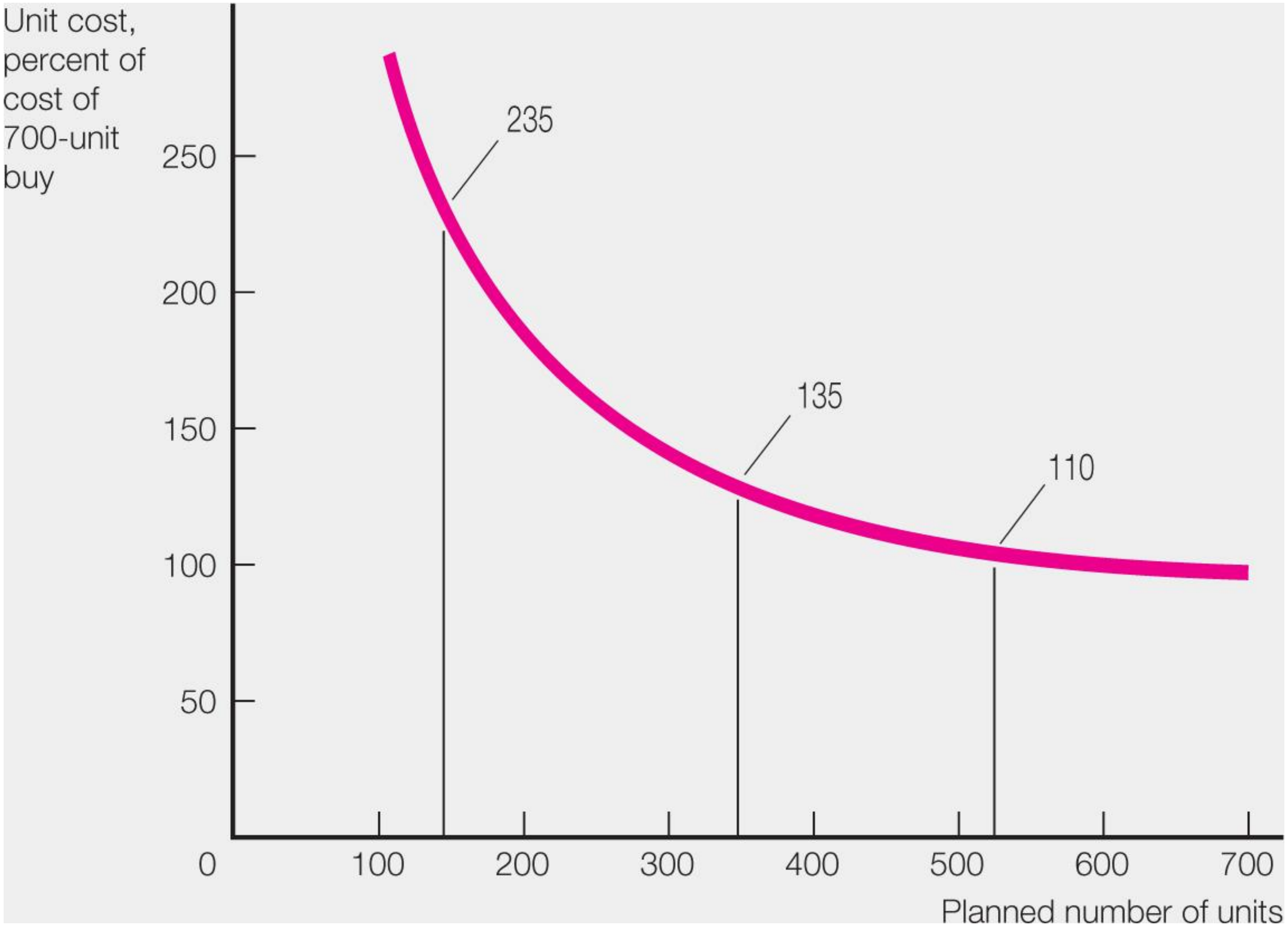
TABLE 6.3

Relationship of Average Product and Marginal Product to Average Variable Cost and Marginal Cost: Thomas Machine Company

L	Q	AP_L	$MP_L = dQ/dL$	W	$AVC = W/AP_L$	$MC = W/MP_L$
0	0	—	—	390	—	—
1	49	49	67	390	7.96	5.82
2	132	66	98	390	5.91	3.98
3	243	81	123	390	4.81	3.17
4	376	94	142	390	4.15	2.75
5	525	105	155	390	3.71	2.52
6	684	114	162	390	3.42	2.41
6.67	792.6	118.9	163.33	390	3.28	2.388 ← MP_L max
7	847	121	163	390	3.22	2.393 so MC min
8	1008	126	158	390	3.10	2.47
9	1161	129	147	390	3.02	2.65
10	1300	130	130	390	3.00 ← AP_L max	3.00
11	1419	129	107	390	3.02 so AVC min	3.64
12	1512	126	78	390	3.10	5.00
13	1573	121	43	390	3.22	9.07
14	1596	114	2	390	3.42	195.00
15	1575	105	−45	390	3.71	—

THE EFFECTS OF OUTPUT ON THE COST OF PRODUCING AIRCRAFT



Long run
Planning period with complete input flexibility.

Long-run cost function
Basis for long-range planning.

Long run cost curves
Cost-output relation for the optimal plant in the present operating environment.

In the long run there are no fixed cost, all cost are variable.(LRTC=LRVC)

However the long run variable cost function will incorporate different quantities of short run fixed costs. For instance maybe there was a small plant that got rebuilt into a bigger plant. Both production processes should be captured in the long run variable (total) cost.

LONG-RUN COST FUNCTIONS

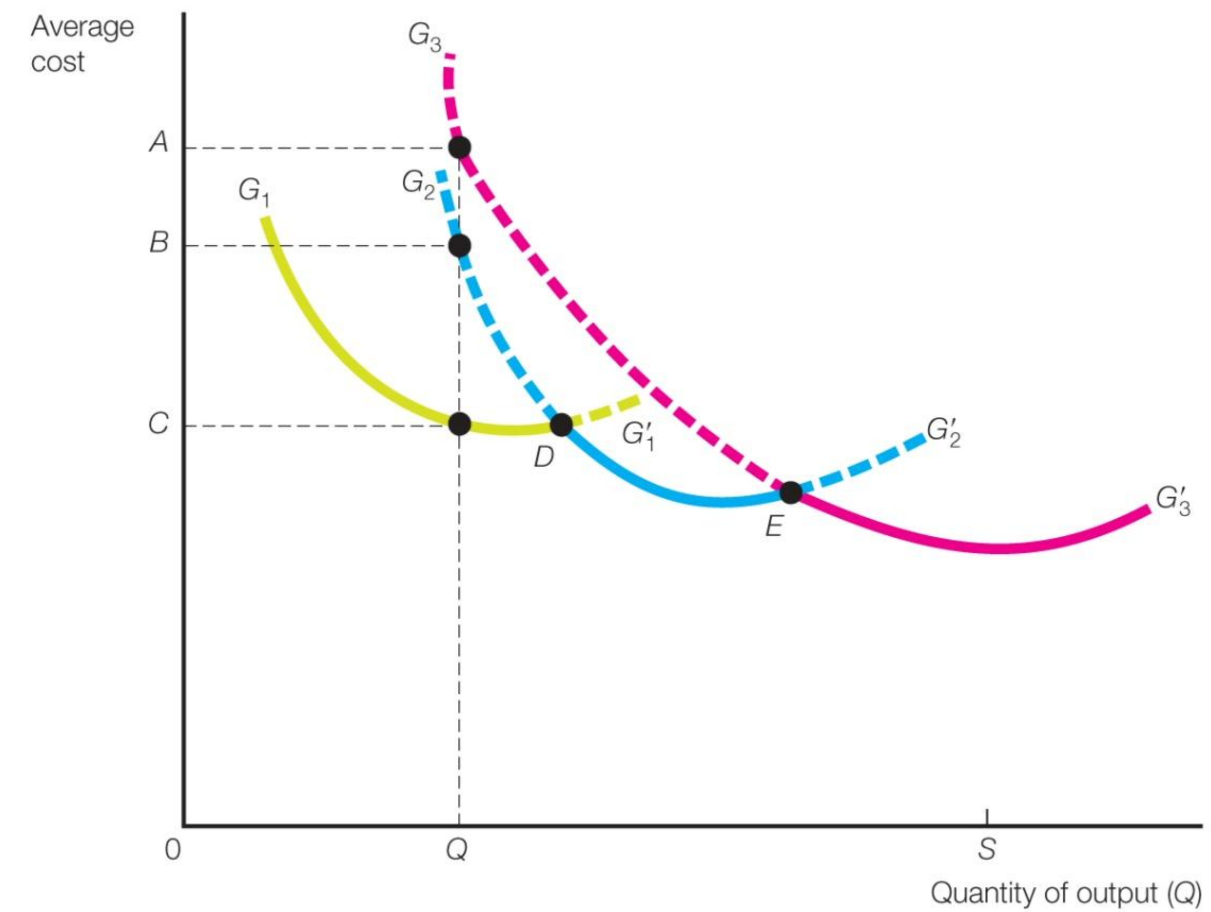
- Definitions
  - Long-run total cost function: Relationship between long-run total cost and output
  - Long-run average cost function: Function showing the minimum cost per unit of all output levels when any desired size plant is built
  - Long-run marginal cost function: Function representing how varying output affects the cost of producing the last unit if the manager has chosen the most efficient input bundle

Minimizing SR cost curves represent the "optimal" (perhaps profit maximizing) level given fixed cost. LR cost curve will give the optimal level without fixed cost. We can think of the LR cost curve therefore, as the collection of optimal point on the short run cost curve with different combinations of fixed cost (like plant and equipment).

SHORT-RUN AVERAGE COST FUNCTIONS FOR VARIOUS SCALES OF PLANT

FIGURE 6.3

Short-Run Average Cost Functions for Various Scales of Plant



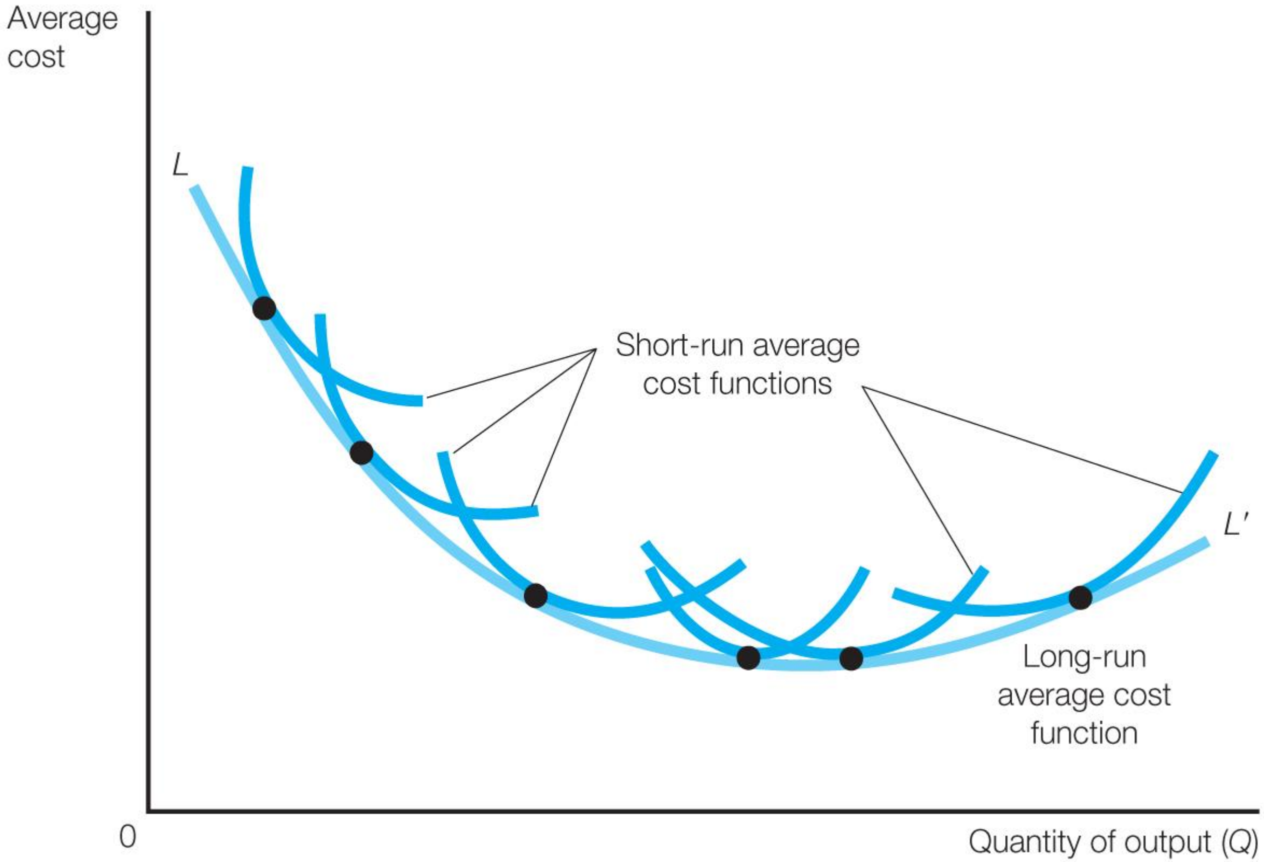
If we think about the average cost curves with the LRAC curves being tangent to the group of SRAC curves.

So the LRAC curve is the “Envelope” of the SRAC curves.

LONG-RUN AVERAGE COST FUNCTION

FIGURE 6.4

Long-Run Average Cost Function



Capacity

Output level at which short-run average costs are minimized.

Minimum efficient scale (MES)

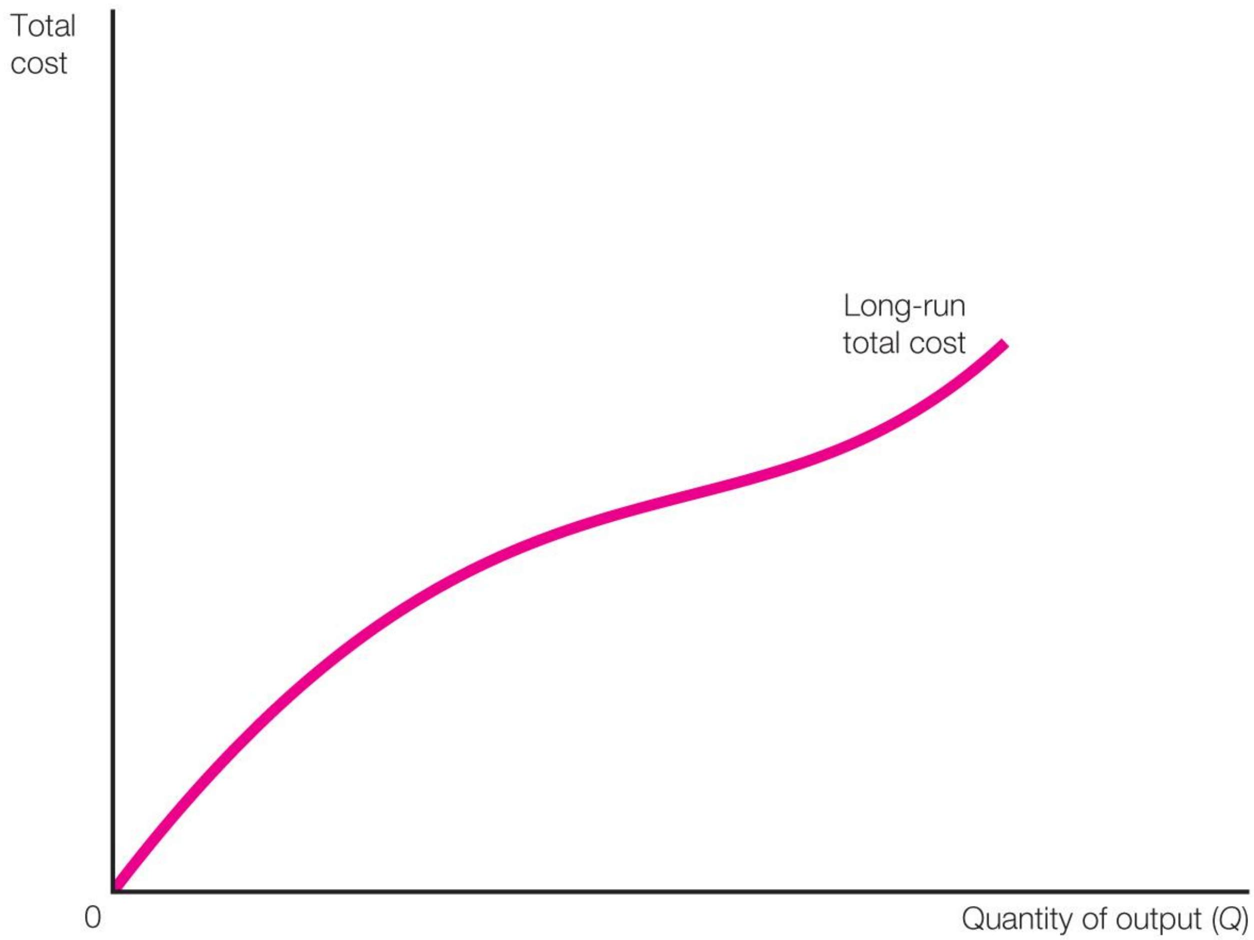
Output level at which long-run costs are minimized.

The MES of the typical firm in a market will determine the market structure. If cost are minimized at high levels of output there is likely to be little competition. If cost are minimized at low levels of output there is likely to be a lot of competition.

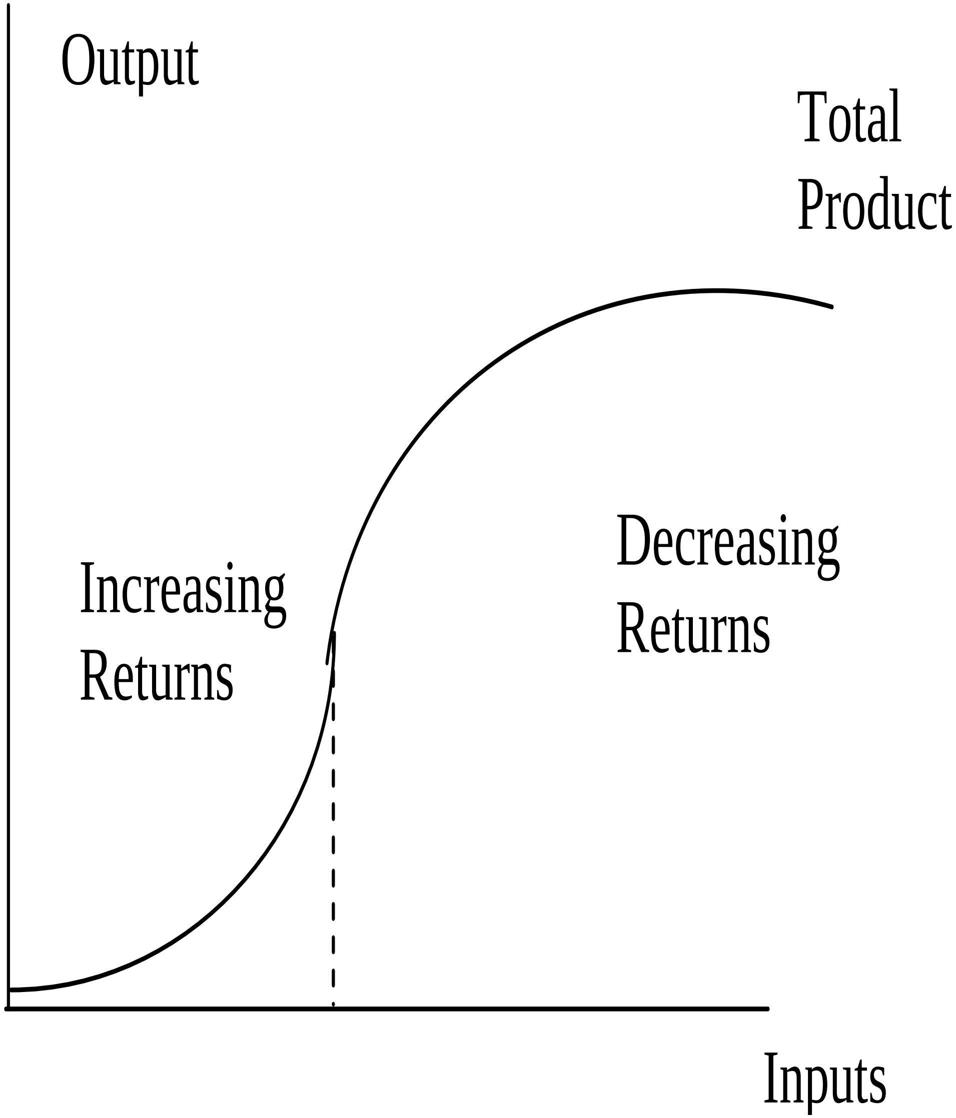
LONG-RUN TOTAL COST FUNCTION

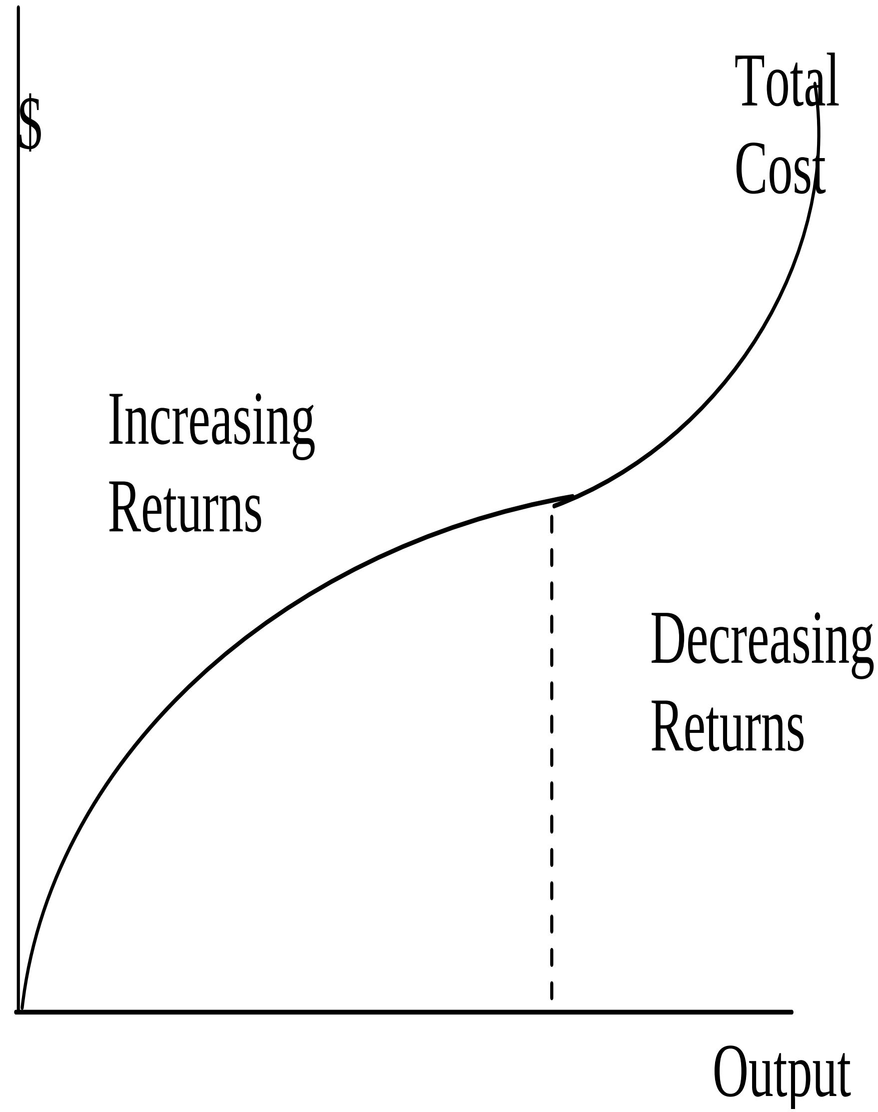
FIGURE 6.5

Long-Run Total Cost Function



Economies of Scale
Decreasing (usually long run) average costs. Equivantly this increasing output per unit of input. In other word each unit of combined input results in more output than the unit before it did. In the graphs below is what economist believe the typical cost and production function look like. They start with increasing returns then decreasing returns to scale. (Right were increasing returns turn to decreasing returns we have constant returns to scale.)


Output
Total
Product
Increasing
Returns
Decreasing
Returns
Inputs


$
Total
Cost
Increasing
Returns
Decreasing
Returns
Output

If we were to put the production graph on it side where both graphs have output on the same axis the change from increasing returns to decreasing return should be the same. (Assuming constant prices of inputs.)

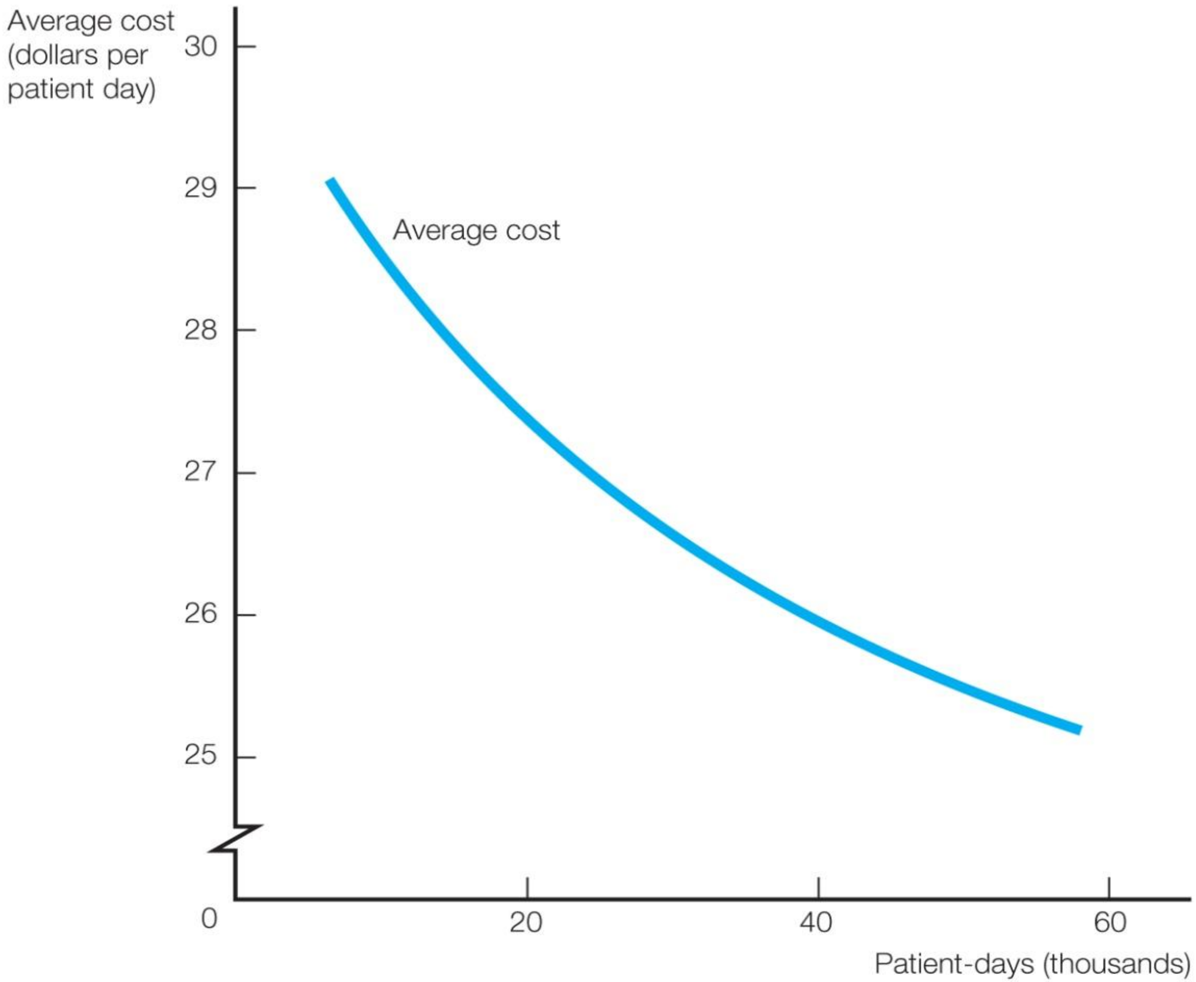
MANAGERIAL USE OF SCALE ECONOMIES

- Economies of scale: When the firm's average unit cost decreases as output increases
    - Economies of scale at Texas nursing homes
    - Economies of scale can result from larger plant size and/or an increase in the number of plants.
    - Economies of scale may be exploited by changes in production, distribution, raising capital, advertising, and other business processes.
- Diseconomies of scale: When the average unit costs increase as output increases

LONG-RUN AVERAGE COST CURVE FOR TEXAS NURSING HOMES

FIGURE 6.6

Long-Run Average Cost Curve for Texas Nursing Homes



Multiplant economies of scale
Cost advantages from operating multiple facilities in the same line of business or industry.

Multiplant diseconomies of scale
Cost disadvantages from managing multiple facitlties in the same line of business or industry.

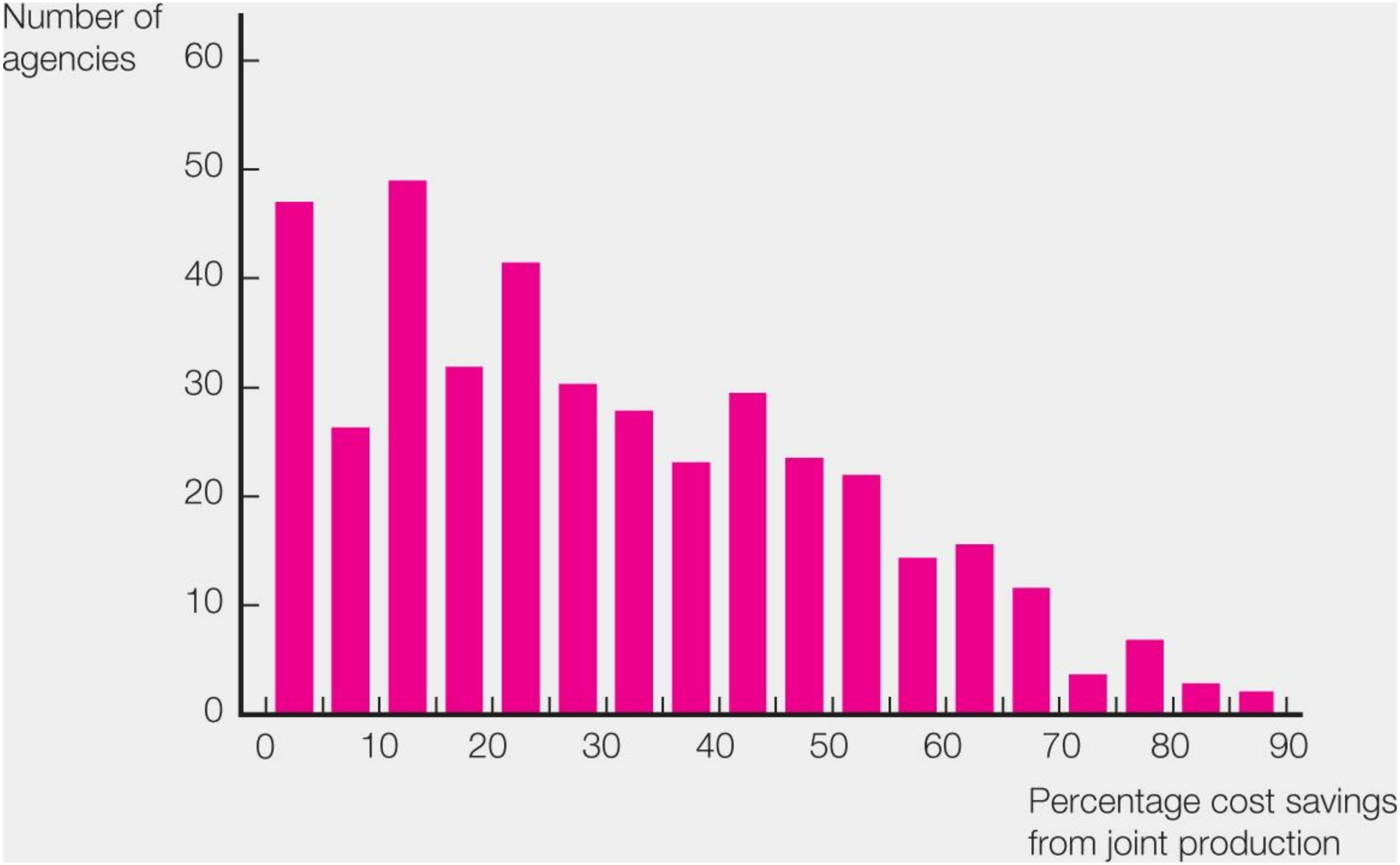
Learning curve
Average cost reduction over time due to production experience.

Economies of scope
Cost reduction for producing complementary products

Cost-volume-profit analysis
Analytical technique used to study relations among cost, revenues and profits.

MANAGERIAL USE OF SCOPE ECONOMIES

- Economies of scope exist when the cost of jointly producing two (or more) products is less than the cost of producing each one alone.
- $$S = \frac{C(Q_1) + C(Q_2) - C(Q_1 + Q_2)}{C(Q_1 + Q_2)}$$
  - S = Degree of economies of scope
  - $C(Q_1)$ = Cost of producing Q_1 units of product 1
  - $C(Q_2)$ = Cost of producing Q_2 units of product 2
  - $C(Q_1 + Q_2)$ = Cost of producing Q_1 units of product 1 and Q_2 units of product 2



Number of agencies
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
Percentage cost savings from joint production

THE NETWORK FOR A HYPOTHETICAL FEDERAL EXPRESS

FIGURE 6.7

The Network for a Hypothetical Federal Express

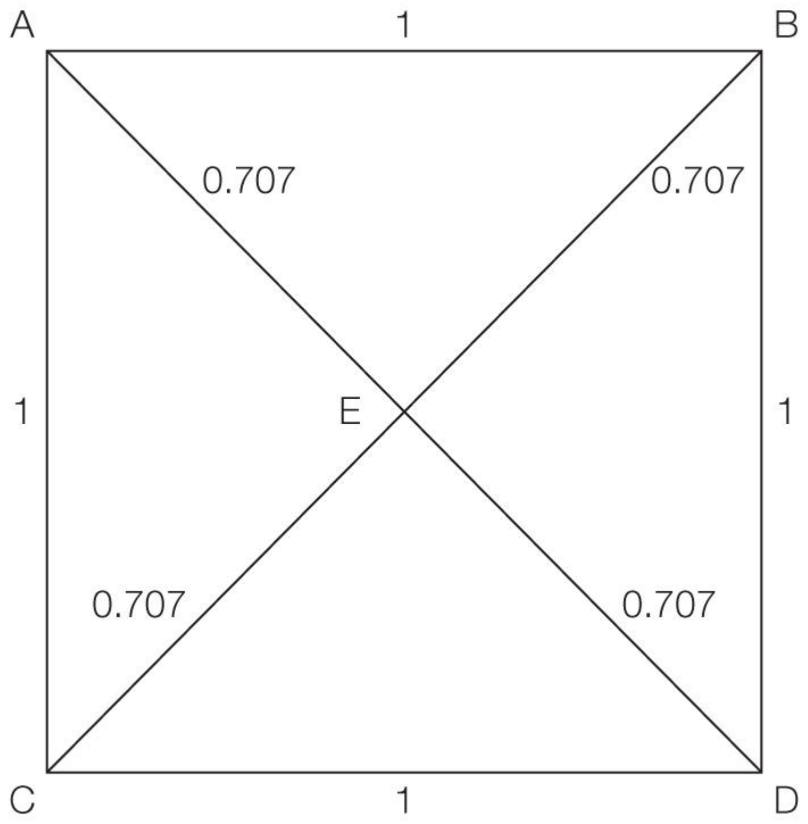


TABLE 6.4

Operating Cost if Each City Pair was Served in Point-to-Point Service

Route	Operating Cost
A to B	0.75 = 1 × 0.75
B to A	0.75
A to C	0.75
C to A	0.75
A to D	1.0605 = 0.75 × 1.414
D to A	1.0605
A to E	0.53025 = 0.75 × 0.707
E to A	0.52025
B to C	1.0605
C to B	1.0605
B to D	0.75
D to B	0.75
B to E	0.53025
E to B	0.53025
C to D	0.75
D to C	0.75
C to E	0.53025
E to C	0.53025
D to E	0.53025
E to D	0.53025
Total	**14.484**

TABLE 6.5

Operating Costs if Each City is Served Via a Hub at E

Flight	Cost
A to E	0.707 = 1 × 0.707
E to A	0.707
B to E	0.707
E to B	0.707
C to E	0.707
E to C	0.707
D to E	0.707
E to D	0.707
Total	**5.656**

Total revenue:

R(q) = P(q) *q

Marginal revenue:

$$MR(q) = \frac{dR}{dq} = p\frac{dq}{dq} + q\frac{dp}{dq} = p + q\frac{dp}{dq}$$

Π = Total revenue – Total cost = R(q) – C(q)

Max. Profit:

$$\frac{\partial \Pi}{\partial q} = \frac{dR}{dq} - \frac{dC}{dq} = 0 \Rightarrow \frac{dR}{dq} = \frac{dC}{dq}$$

MR = MC at profit maximization.

MR and Elasticity of Demand

$$MR = p + q\frac{dp}{dq} = p + \frac{p}{p}q\frac{dp}{dq} = p\left[1 + \frac{q}{p}\frac{dp}{dq}\right]$$

$$MR = p\left[1 + \frac{1}{\frac{p}{q}\frac{dq}{dp}}\right] = p\left[1 + \frac{1}{e_{p,q}}\right]$$

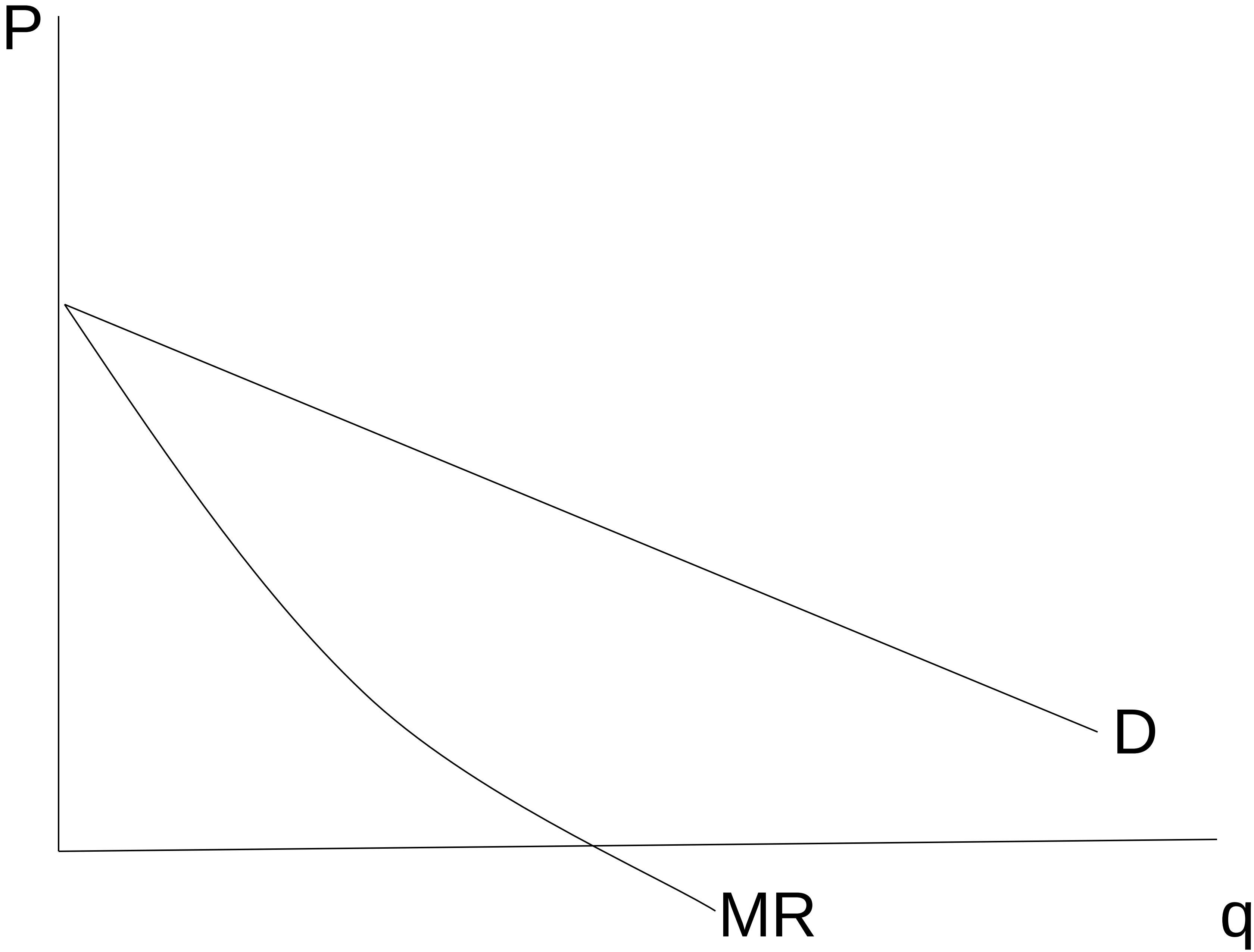
$$e_{p,q} < -1 \Rightarrow MR > 0$$

$$e_{p,q} = -1 \Rightarrow MR = 0$$

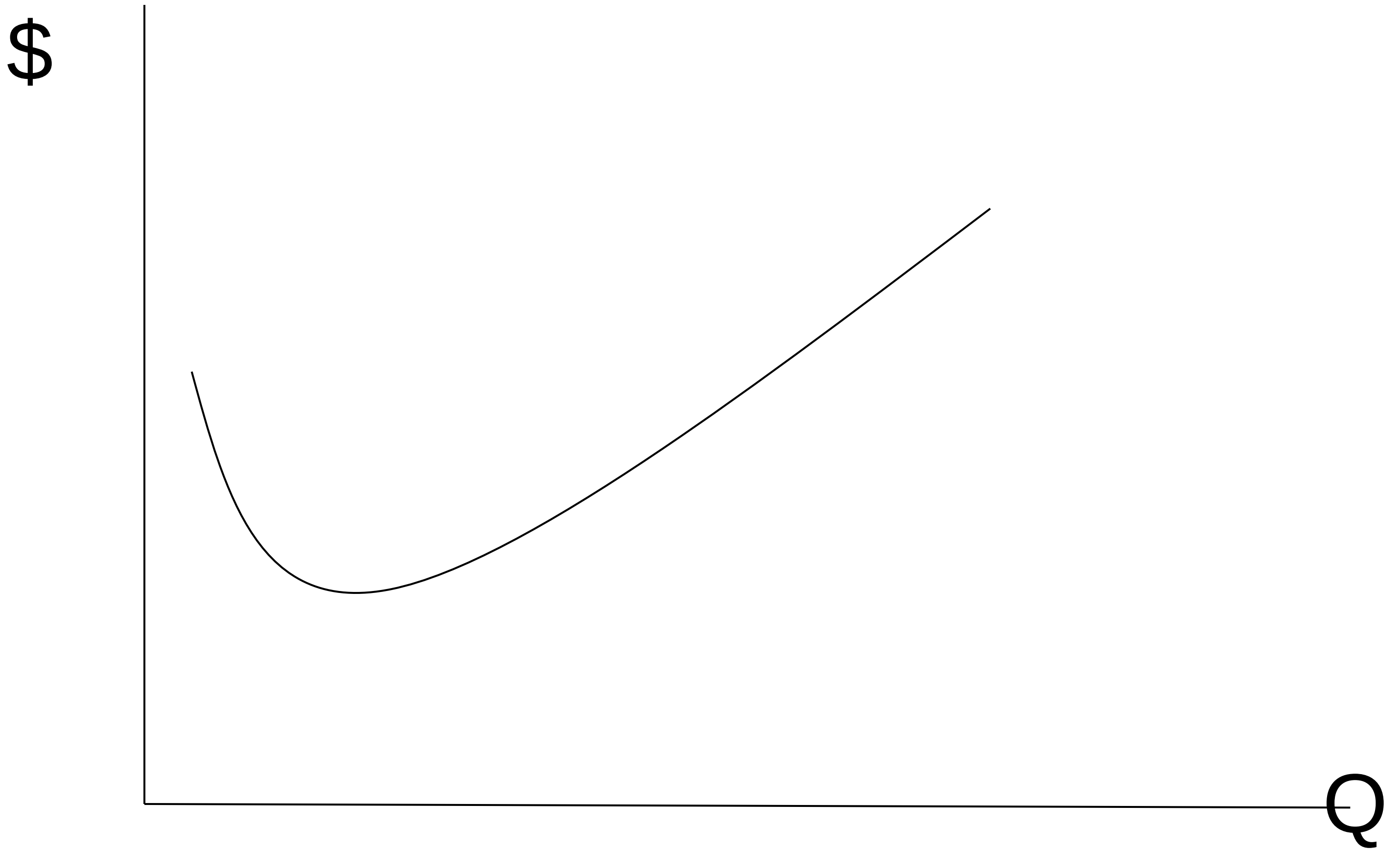
$$e_{p,q} > -1 \Rightarrow MR < 0$$

At profit maximization: MR = MC so:

$$MC = p\left[1 + \frac{1}{e_{q,p}}\right] \Rightarrow \frac{P - MC}{p} = -\frac{1}{e_{q,p}}$$


P
D
MR
q

Firm's short run supply curve for price taking firm, the upper slope portion of MC.



The sum of all MC will be the industry supply curve.

Profit function:

$\Pi = Pq - C = Pf(K,L) - vK - wL$

$\Pi(P,v,w) = Max_{K,q}\,\Pi(K,L) = Max_{K,L}[Pf(K,L) - vK - wL]$

Properties of Π function:

1. Homogeneity in prices (Input & Output)
2. Profit functions are non-decreasing in output prices.
3. Profit functions are non-increasing in input price.
4. Profit functions are convex in output prices.

Marginal Revenue Product

Def: The extra revenue the firm receives when it employs one more unit of input.

In price taking case, where P is constant:

$MRP_L = MP_L * P = P f_L$

$MRP_K = MP_K * P = P f_K$

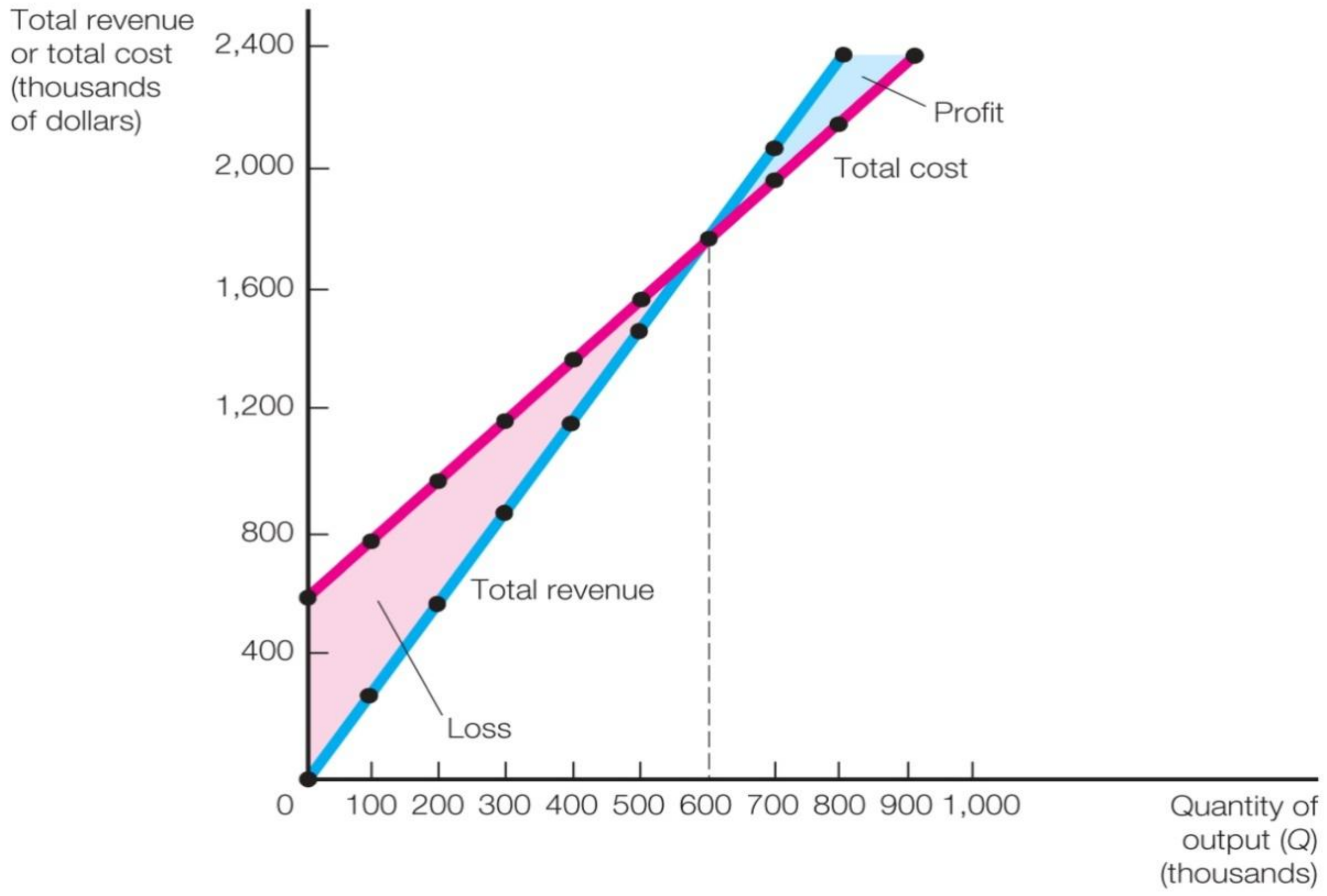
MANAGERIAL USE OF BREAK-EVEN ANALYSIS

- Break-even point (Q_B): Output level that must be reached if managers are to avoid losses
- $Q_B = TFC/(P - AVC)$
  - TFC = Total fixed cost
  - P = Price
  - AVC = Average variable cost
- Example
  - TFC = $600,000
  - P = $3
  - AVC = $2
  - Q_B = 600,000

BREAK-EVEN CHART: MARTIN COMPANY

FIGURE 6.8

Break-Even Chart: Martin Company



PROFIT CONTRIBUTION ANALYSIS

- Profit contribution analysis: a break-even analysis to understand the relationship between price and profit
- $Q_{B'}$ = (TFC + Profit target)/(P – AVC)
  - $Q_{B'}$ = Minimum output level that will attain the profit target

PROFIT CONTRIBUTION ANALYSIS

- Profit contribution analysis (cont'd)
- Example
  - $Q_{B'}$ = (\$600,000 + \$1,000,000)/(\$3 – \$2) = 1,600,000
  - TFC = \$600,000
  - Profit target = \$1,000,000
  - Price = \$3
  - AVC = \$2