Chapter 8

Principles of Pricing Forwards, Futures, and Options on Futures

Chapter 8: Principles of Pricing Forwards, Futures, and Options on Futures

Even if we didn't believe it for a second, there's an undeniable adrenaline jab that comes from someone telling you that you're going to make five hundred million dollars.

Doyne Farmer

Quoted in *The Predictors*, 1999, Page 119.

Important Concepts in Chapter 8

- Price and value of forward and futures contracts
- Relationship between forward and futures prices
- Determination of the spot price of an asset
- Carry arbitrage model for theoretical fair price
- Contango, backwardation, and convenience yield
- Futures prices and risk premiums
- Pricing options on futures

Key Assumptions

- Forward contracts are not subject to margin requirements
- Forward contracts are not centrally cleared
- Forward contracts are not otherwise guaranteed by a third party.
- For forward contracts, the risk of default is so small as to be irrelevant.

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Generic Carry Arbitrage (1 of 5)

- The Concept of Price Versus Value
  - Normally in an efficient market, price = value.
  - For a futures or forward, price is the contracted rate of future purchase. Value is something different.
  - At the beginning of a contract, value = 0 for both futures and forwards.
- Notation
  - $V_t(0,T)$, $F(0,T)$, $v_t(T)$, $f_t(T)$ are values and prices of forward and futures contracts created at time 0 and expiring at time T.

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Generic Carry Arbitrage (2 of 5)

- The Value of a Forward Contract
  - Forward price at expiration:
    - $F(T,T) = S_T$.
    - That is, the price of an expiring forward contract is the spot price.
  - Value of forward contract at expiration:
    - $V_T(0,T) = S_T - F(0,T)$.
    - An expiring forward contract allows you to buy the asset, worth S_T, at the forward price $F(0,T)$. The value to the short party is (−1) times this.

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Generic Carry Arbitrage (3 of 5)

- The Value of a Forward Contract (continued)
  - The Value of a Forward Contract Prior to Expiration
    - A: Go long forward contract at price F(0,T) at time 0.
    - B: At time t go long the asset and take out a loan promising to pay F(0,T) at T
      - At time T, A and B are worth the same, $S_T - F(0,T)$. Thus, they must both be worth the same prior to T.
      - So $V_t(0,T) = S_t - F(0,T)(1 + r)^{-(T-t)}$
      - See Table 8.1 on 32nd slide.
    - Example: Go long 45 day contract at F(0,T) = \$100. Risk-free rate = 0.10. 20 days later, the spot price is \$102. The value of the forward contract is $102 - 100(1.10)^{-25/365} = 2.65$.

Generic Carry Arbitrage (4 of 5)

- The Value of a Futures Contract
  - Futures price at expiration:
    - $f_T(T) = S_T$.
  - Value during the trading day but before being marked to market:
    - $v_t(T) = f_t(T) - f_{t-1}(T)$.
  - Value immediately after being marked to market:
    - $v_t(T) = 0$.

Generic Carry Arbitrage (5 of 5)

- Forward Versus Futures Prices
  - Forward and futures prices will be equal
    - One day prior to expiration
    - More than one day prior to expiration if
      - Interest rates are certain
      - Futures prices and interest rates are uncorrelated
  - Futures prices will exceed forward prices if futures prices are positively correlated with interest rates.
  - Default risk can also affect the difference between futures and forward prices.

Carry Arbitrage: Equities (1 of 3)

- Forward and Futures Pricing When the Underlying Generates Cash Flows
  - For example, dividends on a stock or index
    - Assume one dividend D_T paid at expiration.
    - Buy stock, sell futures guarantees at expiration that you will have $D_T + f_0(T)$. Present value of this must equal S_0, using risk-free rate. Thus,
      - $f_0(T) = S_0(1+r)^T - D_T$.
    - For multiple dividends, let D_T be compound future value of dividends. See Figure 8.1 on 33rd slide for two dividends.
    - Dividends reduce the cost of carry.
    - If D_0 represents the present value of the dividends, the model becomes
      - $f_0(T) = (S_0 - D_0)(1+r)^T$.

Carry Arbitrage: Equities (2 of 3)

- Forward and Futures Pricing When the Underlying Generates Cash Flows (continued)
  - For dividends paid at a continuously compounded rate of δ_c,

$$f(0,T) = S_0 e^{(r_c - \delta_c)T}$$

  - Example: S0 = 50, r_c = 0.08, δ_c = 0.06, expiration in 60 days $(T = 60/365 = 0.164)$.
    - $f_0(T) = 50e^{(0.08\ -\ 0.06)(0.164)} =\ 50.16.$

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Carry Arbitrage: Equities (3 of 3)

- Valuation of Equity Forward Contracts
  - When there are dividends, to determine the value of a forward contract during its life
    - $V_t(0,T) = S_t - D_{t,T} - F(0,T)(1+r)^{-(T-t)}$
    - where $D_{t,T}$ is the value at time t of the future dividends to time T
  - Or if dividends are continuous,

$$V_t(0,T) = S_t e^{-\delta_c (T-t)} - F(0,T)e^{-r_c (T-t)}$$

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Carry Arbitrage: Currencies (1 of 5)

- Pricing Foreign Currency Forward and Futures Contracts: Interest Rate Parity
  - Interest Rate Parity: the relationship between futures or forward and spot exchange rates. Same as carry arbitrage model in other forward and futures markets.
  - Proves that one cannot convert a currency to another currency, sell a futures, earn the foreign risk-free rate, and convert back without risk, earning a rate higher than the domestic rate.

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Carry Arbitrage: Currencies (2 of 5)

- Pricing Foreign Currency Forward and Futures Contracts: Interest Rate Parity (continued)
  - S_0 = spot rate in domestic currency per foreign currency. Foreign rate is ρ. Holding period is T. Domestic rate is r.
    - Take $S_0(1+\rho)^{-T}$ units of domestic currency and buy $(1+\rho)^{-T}$ units of foreign currency.
    - Sell forward contract to deliver one unit of foreign currency at T at price F(0,T).
    - Hold foreign currency and earn rate ρ. At T you will have one unit of the foreign currency.
    - Deliver foreign currency and receive F(0,T) units of domestic currency.

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Carry Arbitrage: Currencies (3 of 5)

- Pricing Foreign Currency Forward and Futures Contracts: Interest Rate Parity (continued)
  - So an investment of $S_0(1+\rho)^{-T}$ units of domestic currency grows to F (0,T) units of domestic currency with no risk. Return should be r. Therefore
    - $F(0,T) = S_0(1+\rho)^{-T}(1+r)^{T}$
  - This is called interest rate parity.
  - Sometimes written as
    - $F(0,T) = S_0(1+r)^{T}/(1+\rho)^{T}$

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Carry Arbitrage: Currencies (4 of 5)

- Pricing Foreign Currency Forward and Futures Contracts: Interest Rate Parity (continued)
  - Example (from a European perspective): S_0 = €1.0304. U.S. rate is 5.84%. Euro rate is 3.59%. Time to expiration is $90/365 = 0.2466$.
    - $F(0,T) = €1.0304(1.0584)^{-0.2466}(1.0359)^{0.2466} = €1.025$
  - If forward rate is actually €1.03, then it is overpriced.
    - Buy $(1.0584)^{-0.2466} =$ \$0.9861 for $0.9861(€1.0304) = €1.0161$. Sell one forward contract at €1.03.
    - Earn 5.84% on \$0.9861. This grows to \$1.
    - At expiration, deliver \$1 and receive €1.03.
    - Return is $(1.03/1.0161)^{365/90} - 1 = 0.0566\ (> 0.0359)$
    - This transaction is called covered interest arbitrage.

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Carry Arbitrage: Currencies (5 of 5)

- Pricing Foreign Currency Forward and Futures Contracts: Interest Rate Parity (continued)
  - It is also sometimes written as
    - $F(0,T) = S_0(1+\rho)^T(1+r)^{-T}$
    - Here, the spot rate is being quoted in units of the foreign currency.
  - Note that the forward discount/premium has nothing to do with expectations of future exchange rates.
  - Difference between domestic and foreign rate is analogous to difference between risk-free rate and dividend yield on stock index futures.

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Pricing Models and Risk Premiums (1 of 5)

- Spot Prices, Risk Premiums, and the Carry Arbitrage for Generic Assets
  - First assume no uncertainty of future price. Let s be the cost of storing an asset and i be the interest rate for the period of time the asset is owned. Then
    - $S_0 = S_T - s - iS_0$
  - If we now allow uncertainty but assume people are risk neutral, we have
    - $S_0 = E(S_T) - s - iS_0$
  - If we now allow people to be risk averse, they require a risk premium of $E(\Phi)$. Now
    - $S_0 = E(S_T) - s - iS_0 - E(\Phi)$

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Pricing Models and Risk Premiums (2 of 5)

- Spot Prices, Risk Premiums, and the Carry Arbitrage for Generic Assets (continued)
  - Let us define iS_0 as the net interest, which is the interest foregone minus any cash received.
  - Define $s + iS_0$ as the cost of carry.
  - Denote cost of carry as θ.
  - Note how cost of carry is a meaningful concept only for storable assets

Pricing Models and Risk Premiums (3 of 5)

- The Theoretical Fair Price (Forward/Futures Pricing Revisited)
  - Do the following
    - Buy asset in spot market, paying S_0; sell futures contract at price $f_0(T)$; store and incur costs.
    - At expiration, make delivery. Profit:
      - $\Pi = f_0(T) - S_0 - \theta$
  - This must be zero to avoid arbitrage; thus,
      - $f_0(T) = S_0 + \theta$
  - See Figure 8.2 on 34th slide.
  - Note how arbitrage and quasi-arbitrage make this hold.

Pricing Models and Risk Premiums (4 of 5)

- Forward/Futures Pricing Revisited (continued)
  - See Figure 8.3 on 35th slide, for an illustration of the determination of futures prices.
  - Contango is $f_0(T) > S_0$. See Table 8.2 on 36th slide.
  - When $f_0(T) < S_0$, convenience yield is χ, an additional return from holding asset when in short supply or a non-pecuniary return. Market is said to be at less than full carry and in backwardation or inverted. See Table 8.3 on 37th slide. Market can be both backwardation and contango. See Table 8.4 on 38th slide.

Pricing Models and Risk Premiums (5 of 5)

- Futures Prices and Risk Premia
  - The no risk-premium hypothesis
    - Market consists of only speculators.
    - $f_0(T) = E(S_T)$. See Figure 8.4 on 39th slide.
  - The risk-premium hypothesis
    - $E(f_T(T)) > f_0(T)$.
    - When hedgers go short futures, they transfer risk premium to speculators who go long futures.
    - $E(S_T) = f_0(T) + E(\Phi)$. See Figure 8.5 on 40th slide.
  - Normal contango: $E(S_T) < f_0(T)$
  - Normal backwardation: $f_0(T) < E(S_T)$

Put-Call-Forward/Futures Parity (1 of 2)

- Can construct synthetic futures with options.
- See Table 8.5 on 41st slide.
- Put-call-forward/futures parity
  - $P_e(S_0,T,X) = C_e(S_0,T,X) + \left[X - f_0(T)\right](1+r)^{-T}$
- Numerical example using S&P 500. On May 14, S&P 500 at 1337.80 and June futures at 1339.30. June 1340 call at 40 and put at 39. Expiration of June 18 so T = 35/365 = 0.0959. Risk-free rate at 4.56%.

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Put-Call-Forward/Futures Parity (2 of 2)

- So $P_e(S_0,T,X) = 39$
- $C_e(S_0,T,X) + \left[X - f_0(T)\right](1+r)^{-T}$
- $= 40 + (1340 - 1339.30)(1.0456)^{-0.0959} = 40.70.$
- Buy put and futures for 39, sell call and bond for 40.70 and net 1.70 profit at no risk. Transaction costs would have to be considered.

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Pricing Options on Futures (1 of 6)

- The Intrinsic Value of an American Option on Futures
  - Minimum value of American call on futures
    - $C_a(f_0(T),T,X) \geq \text{Max}[0, f_0(T) - X]$
  - Minimum value of American put on futures
    - $P_a(f_0(T),T,X) \geq \text{Max}[0, X - f_0(T)]$
  - Difference between option price and intrinsic value is time value.

Pricing Options on Futures (2 of 6)

- The Lower Bound of a European Option on Futures
  - For calls, construct two portfolios. See Table 8.6 on 42nd slide.
  - Portfolio A dominates Portfolio B so
    - $C_e(f_0(T),T,X) \geq \text{Max}\left\{0,\left[f_0(T) - X\right](1+r)^{-T}\right\}$
  - Note that lower bound can be less than intrinsic value even for calls.
  - For puts, see Table 8.7 on 43rd slide.
  - Portfolio A dominates Portfolio B so
    - $P_e(f_0(T),T,X) \geq \text{Max}\left\{0,\left[X - f_0(T)\right](1+r)^{-T}\right\}$

Pricing Options on Futures (3 of 6)

- Put-Call Parity of Options on Futures
  - Construct two portfolios, A and B.
  - See Table 8.8 on 44th slide.
  - The portfolios produce equivalent results. Therefore they must have equivalent current values. Thus,
    - $P_e(f_0(T),T,X) = C_e(f_0(T),T,X) + \left[X - f_0(T)\right](1+r)^{-T}$.
  - Compare to put-call parity for options on spot:
    - $P_e(S_0,T,X) = C_e(S_0,T,X) - S_0 + X(1+r)^{-T}$.
    - If options on spot and options on futures expire at same time, their values are equal, implying $f_0(T) = S_0(1+r)^{T}$, which we obtained earlier (no cash flows).

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Pricing Options on Futures (4 of 6)

- Early Exercise of Call and Put Options on Futures
  - Deep in-the-money call may be exercised early because
    - behaves almost identically to futures
    - exercise frees up funds tied up in option but requires no funds to establish futures
    - minimum value of European futures call is less than value if it could be exercised
  - See Figure 8.6 on 45th slide.
  - Similar arguments hold for puts
  - Compare to the arguments for early exercise of call and put options on spot.

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Pricing Options on Futures (5 of 6)

- Options on Futures Pricing Models
  - Black model for pricing European options on futures

$$
\begin{aligned}
&C = e^{-r_c T}[f_0(T)N(d_1) - XN(d_2)] \\
&\text{where} \\
&d_1 = \frac{\ln(f_0(T)/X) + \left(\sigma^2/2\right)T}{\sigma\sqrt{T}} \\
&d_2 = d_1 - \sigma\sqrt{T}
\end{aligned}
$$

Pricing Options on Futures (6 of 6)

- Options on Futures Pricing Models (continued)
  - Note that with the same expiration for options on spot as options on futures, this formula gives the same price.
  - Example
    - See Table 8.9 on 46th slide.
  - Software for Black-Scholes-Merton can be used by inserting futures price instead of spot price and risk-free rate for dividend yield. Note why this works.
  - For puts

$$P = Xe^{-r_c T}[1 - N(d_2)] - f_0(T)e^{-r_c T}[1 - N(d_1)]$$

Summary

See Table 8.10 on 47th slide for a summary of equations.

See Figure 8.7 on 48th slide for linkage between forwards/futures, underlying asset and risk-free bond.

TABLE 8.1: VALUING A FORWARD CONTRACT PRIOR TO EXPIRATION

PORTFOLIO	COMPOSITION	VALUE AT 0	VALUE AT t	VALUE AT T
A	Long forward contract established at 0 at price of $F(0,T)$	0	$V_t(0,T)$	$S_T - F(0,T)$
B	Long position in asset and loan of $F(0,T)(1+r)^{-(T-t)}$ established at t	N/A	$S_t - F(0,T)(1+r)^{-(T-t)}$	$S_T - F(0,T)$

Conclusion: The value of portfolio A at t must equal the value of portfolio B at t. $V_t(0,T) = S_t - F(0,T)(1+r)^{-(T-t)}$

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FIGURE 8.1: The Cost of Carry Model with Stock Index Futures

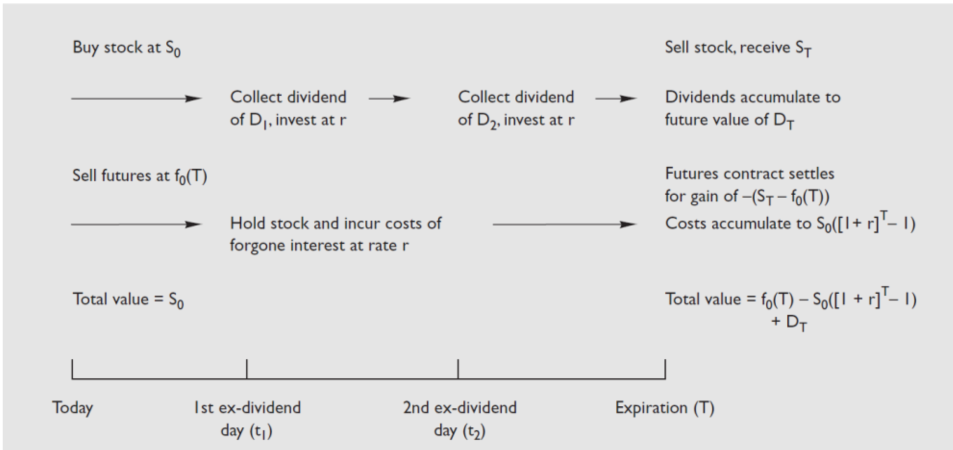

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FIGURE 8.2: Buy Asset, Sell Futures, and Store Asset

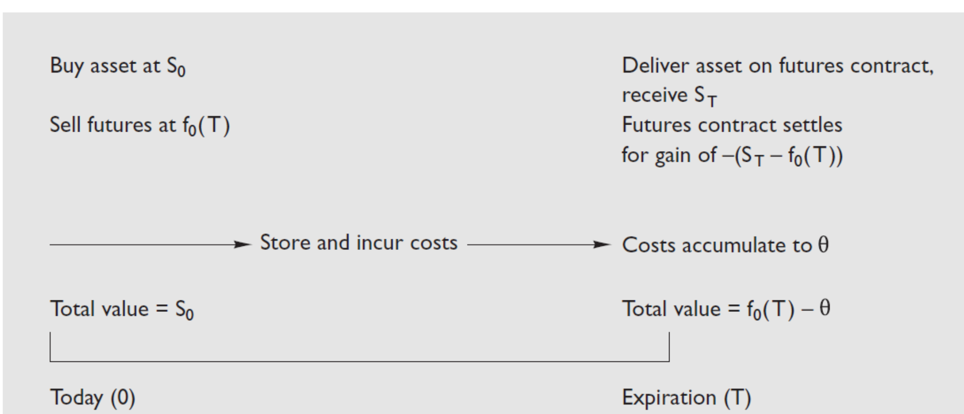

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FIGURE 8.3: How Futures Prices Are Determined

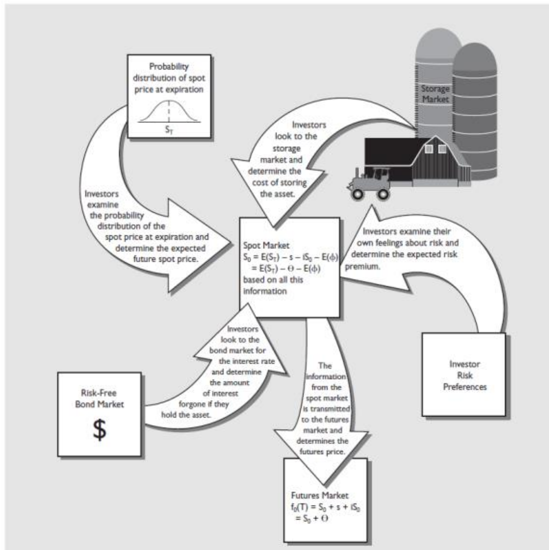

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TABLE 8.2: AN EXAMPLE OF A CONTANGO MARKET

COTTON (INTERCONTINENTAL EXCHANGE)

EXPIRATION	SETTLEMENT PRICE (CENTS PER POUND)
Spot (September 26)	36.75
October	41.60
December	42.05
March	42.77
May	43.50
July	43.80
October	45.20
December	45.85

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TABLE 8.3: AN EXAMPLE OF A BACKWARDATION MARKET

SOYBEANS (CHICAGO BOARD OF TRADE)

EXPIRATION	PRICE (CENTS PER BUSHEL)
November	563.25
January	558.50
March	552.75
May	545.75
July	543.25
August	536.50
September	520.50
November	502.25

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TABLE 8.4: AN EXAMPLE OF A SIMULTANEOUS BACKWARDATION AND CONTANGO MARKET

SOYBEAN MEAL (CHICAGO BOARD OF TRADE)

EXPIRATION	PRICE (CENTS PER BUSHEL)
Spot (November 8)	159.50
December	163.50
January	164.40
March	164.80
May	163.00
July	162.40
August	161.10
September	158.30
October	154.90
December	155.10
January	154.10

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FIGURE 8.4: An Example of No Risk Premium: May Wheat

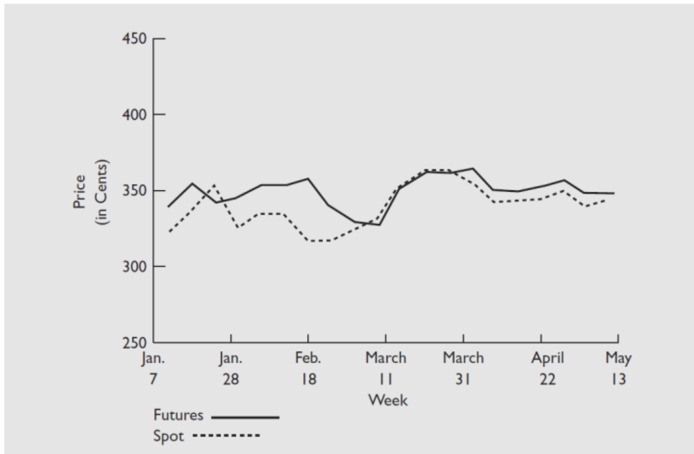

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FIGURE 8.5: An Example of a Risk Premium: S&P 500 Futures and Spot

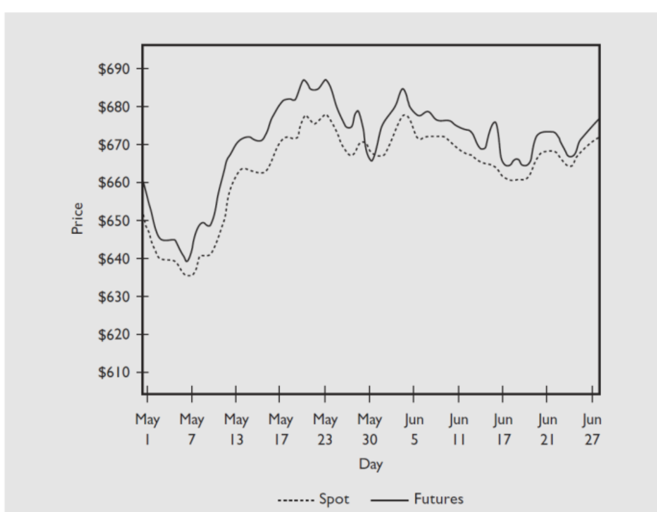

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TABLE 8.5: PUT−CALL−FORWARD/FUTURES PARITY

		PAYOFFS FROM PORTFOLIO GIVEN STOCK PRICE AT EXPIRATION	
PORTFOLIO	CURRENT VALUE	ST $\le$ X	ST $>$ X
A. Futures	0	$S_T - f_0(T)$	$S_T - f_0(T)$
Put	$P_e(S_0,T,X)$	$X - S_T$	0
		$X - f_0(T)$	$S_T - f_0(T)$
B. Call	$C_e(S_0,T,X)$	0	$S_T - X$
Bonds	$[X - f_0(T)](1+r)^{-T}$	$X - f_0(T)$	$X - f_0(T)$
		$X - f_0(T)$	$S_T - f_0(T)$

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TABLE 8.6: THE LOWER BOUND OF A EUROPEAN CALL OPTION ON FUTURES: PAYOFFS AT EXPIRATION OF PORTFOLIOS A AND B

			PAYOFFS FROM PORTFOLIO GIVEN FUTURES PRICE AT EXPIRATION	
PORTFOLIO	INSTRUMENT	CURRENT VALUE	$f_T(T) \le X$	$f_T(T) > X$
A	Long call	$C_e[f_0(T),T,X]$	0	$f_T(T) - X$
B	Long futures	0	$f_T(T) - f_0(T)$	$f_T(T) - f_0(T)$
	Bond	$[f_0(T) - X](1+r)^{-T}$	$+f_0(T) - X$	$+f_0(T) - X$
			$f_T(T) - X$	$f_T(T) - X$

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TABLE 8.7: THE LOWER BOUND OF A EUROPEAN PUT OPTION ON FUTURES: PAYOFFS AT EXPIRATION OF PORTFOLIOS A AND B

			PAYOFFS FROM PORTFOLIO GIVEN FUTURES PRICE AT EXPIRATION	
PORTFOLIO	INSTRUMENT	CURRENT VALUE	$f_T(T) < X$	$f_T(T) \geq X$
A	Long put	$P_e[f_0(T),T,X]$	$X - f_T(T)$	0
B	Short futures	0	$-[f_T(T) - f_0(T)]$	$-[f_T(T) - f_0(T)]$
	Bond	$+[X - f_0(T)](1 + r)^{-T}$	$+X - f_0(T)$	$+X - f_0(T)$
			$X - f_T(T)$	$X - f_T(T)$

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TABLE 8.8: PUT−CALL PARITY OF OPTIONS ON FUTURES

			PAYOFFS FROM PORTFOLIO GIVEN FUTURES PRICE AT EXPIRATION	
PORTFOLIO	INSTRUMENT	CURRENT VALUE	$f_T(T) \leq X$	$f_T(T) > X$
A	Long futures	0	$f_T(T) - f_0(T)$	$f_T(T) - f_0(T)$
	Long put	$P_e[f_0(T),T,X]$	$X - f_T(T)$	0
			$X - f_0(T)$	$f_T(T) - f_0(T)$
B	Long call	$C_e[f_0(T),T,X]$	0	$f_T(T) - X$
	Bonds	$[X - f_0(T)](1 + r)^{-T}$	$X - f_0(T)$	$X - f_0(T)$
			$X - f_0(T)$	$f_T(T) - f_0(T)$

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FIGURE 8.6: American and European Calls and Puts on Futures

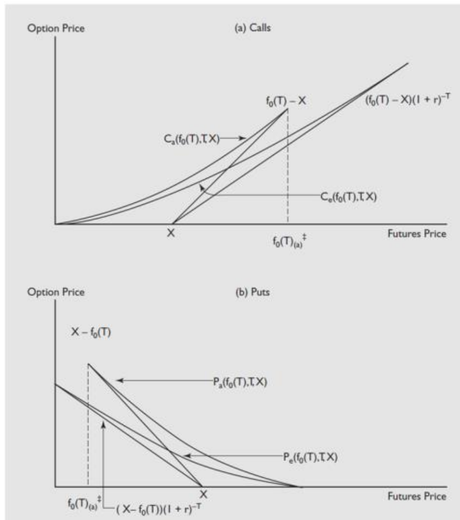

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TABLE 8.9: CALCULATING THE BLACK OPTION ON FUTURES PRICE

$f_0(T) = 1{,}401$	$X = 1{,}400$	$r_c = 0.0543$	$\sigma = 0.21$	$T = 0.1233$
1. Compute d_1	$d_1 = \frac{\ln(1{,}401/1{,}400) + [(0.21)^2/2]0.1233}{0.21\sqrt{0.1233}} = 0.0466$			
2. Compute d_2	$d_2 = 0.0466 - 0.21\sqrt{0.1233} = -0.0271$			
3. Look up $N(d_1)$	$N(0.05) = 0.5199$			
4. Look up $N(d_2)$	$N(-0.03) = 0.4880$			
5. Plug into formula for C	$C = e^{-(0.0543)(0.1233)}[1{,}401(0.5199) - 1{,}400(0.4880)] = 44.88$			

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TABLE 8.10: SUMMARY OF THE PRINCIPLES OF FORWARD/FUTURES VALUE AND PRICING

	FORWARD CONTRACTS	FUTURES CONTRACTS
Pricing equations		
At expiration	$F(T,T) = S_T$	$f_T(T) = S_T$
Prior to expiration	$F(0,T) = S_0 + \theta - \chi$	$f_0(T) = S_0 + \theta - \chi$
Risk premium	$F(0,T) = E(S_T) - E(\phi)$	$f_0(T) = E(S_T) - E(\phi)$
Dividends (discrete)	$F(0,T) = S_0(1+r)^T - D_T$ $F(0,T) = (S_0 - D_0)(1+r)^T$	$f_0(T) = S_0(1+r)^T - D_T$ $f_0(T) = (S_0 - D_0)(1+r)^T$
Dividends (continuous)	$F(0,T) = S_0 e^{(r_c - \delta_c)T}$	$f_0(T) = S_0 e^{(r_c - \delta_c)T}$
Foreign Exchange	$F(0,T) = S_0(1+r)^T/(1+\rho)^T$	$f_0(T) = S_0(1+r)^T/(1+\rho)^T$
Value equations		
At expiration	$V_T(0,T) = S_T - F(0,T)$	$v_t(T) = 0$ (after M2M)
Prior to expiration	$V_t(0,T) = S_t - F(0,T)(1+r)^{-(T-t)}$	$v_t(T) = 0$ (after M2M)
Dividends (discrete)	$V_t(0,T) = S_t - D_{t,T} - F(0,T)(1+r)^{-(T-t)}$	$v_t(T) = 0$ (after M2M)
Dividends (continuous)	$V_t(0,T) = S_t e^{-\delta_c(T-t)} - F(0,T)e^{-r_c(T-t)}$	$v_t(T) = 0$ (after M2M)
Foreign Exchange	$V_t(0,T) = S_t(1+\rho)^{-(T-t)} - F(0,T)(1+r)^{-(T-t)}$	$v_t(T) = 0$ (after M2M)
OPTIONS ON FUTURES	**CALL OPTIONS**	**PUT OPTIONS**
Minimum value	≥ 0	≥ 0
Intrinsic value	$C_a[f_0(T),T,X] \geq \text{Max}[0, f_0(T) - X]$	$P_a[f_0(T),T,X] \geq \text{Max}[0, X - f_0(T)]$
Lower bound	$C_e[f_0(T),T,X] \geq \text{Max}\left\{0, [f_0(T) - X](1+r)^{-T}\right\}$ $C_a[f_0(T),T,X] \geq \text{Max}[0, f_0(T) - X]$	$P_e[f_0(T),T,X] \geq \text{Max}\left\{0, [X - f_0(T)](1+r)^{-T}\right\}$ $P_a[f_0(T),T,X] \geq \text{Max}[0, X - f_0(T)]$
Other results		
Black OPM	$C = e^{-r_c T}[f_0(T)N(d_1) - XN(d_2)]$ $d_1 = \dfrac{\ln[f_0(T)/X] + (\sigma^2/2)T}{\sigma\sqrt{T}}$ $d_2 = d_1 - \sigma\sqrt{T}$	$P = Xe^{-r_c T}[1 - N(d_2)] - f_0(T)e^{-r_c T}[1 - N(d_1)]$
Put-call parity	$C_e[f_0(T),T,X] = P_e[f_0(T),T,X] - [X - f_0(T)](1+r)^{-T}$	$P_e[f_0(T),T,X] = C_e[f_0(T),T,X] + [X - f_0(T)](1+r)^{-T}$
Effect of dividends	No direct effect, forward price lowered when dividends increase	No direct effect, futures price lowered when dividends increase
Currency options	No direct effect, forward price lowered when foreign interest rates increase	No direct effect, futures price lowered when foreign interest rates increase

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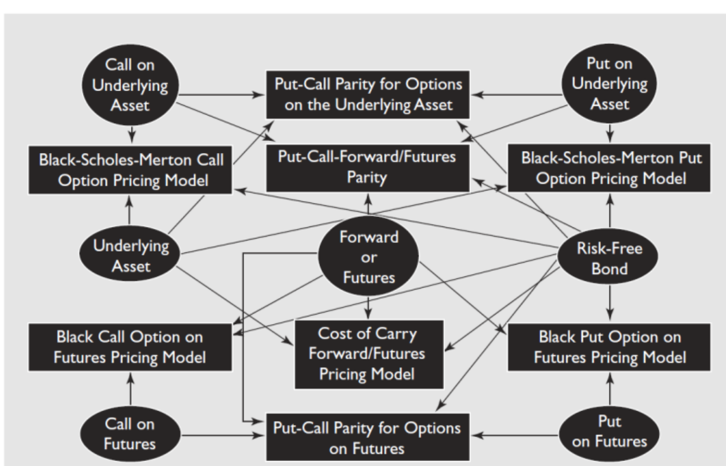
FIGURE 8.7: The Linkage between Forwards/Futures, Stock, Bonds, Options on the Underlying Asset, and Options on the Futures



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