Chapter 9

Futures Arbitrage Strategies

Chapter 9: Futures Arbitrage Strategies

Often Jadwin had noted the scene, and unimaginative though he was, had long since conceived the notion of some great, some resistless force within the Board of Trade Building that held the tide of the streets within its grip, alternately drawing it in and throwing it forth. Within there, a great whirlpool, a pit of roaring water spun and thundered, sucking in the life tides of the city, sucking them in as into the mouth of some tremendous cloaca, the maw of some colossal sewer; the vomiting them forth again, spewing them up and out, only to catch them in the return eddy and suck them in afresh.

Frank Norris

The Pit, 1902, 1994 edition, Penguin Books, p. 72

Important Concepts in Chapter 9

- Futures arbitrage strategies
  - Short-term interest rate futures
  - Long-term interest rate futures
  - Stock index futures
  - Foreign exchange futures
- Cheapest-to-deliver bond
- Delivery options

Short-Term Interest Rate Arbitrage (1 of 2)

- Cash and Carry/Implied Repo
  - Cash and carry transaction means to buy asset and sell futures
  - Repurchase agreement/repo to obtain funding
  - Overnight versus term repo
  - Cost of carry pricing model: $f_0(t) = S_0 + \theta$
  - Implied repo rate:

$$\hat{r} = \left(\frac{f_0(t)}{S_0}\right)^{1/t} - 1$$

Short-Term Interest Rate Arbitrage (2 of 2)

- Cash and Carry/Implied Repo Rate
  - Also equivalent to buying longer term and converting it to shorter term.
  - Example. See Table 9.1. on 35th slide
- Eurodollar Arbitrage
  - Using Eurodollar futures with spot to earn an arbitrage profit.
  - See Table 9.2. on 37th slide

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (1 of 20)

- Recall the option to deliver any T-bond with at least 15 years to maturity or first call.
- Adjustment to futures price using conversion factor, which is the price per $1.00 par of a 6% bond delivered on a particular expiration.
- Invoice price = (Settlement price on position day)(Conversion factor) + Accrued interest
- Example: Delivery on March YY contract. Settlement price is 140 ($140,000) on position day.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (2 of 20)

- You plan to deliver the 5 1/2 of YY+16 on March 8. CF = 0.9485. Coupon dates of February 15 and August 15. Last coupon on February 15, 2012. Days from 2/15 to 3/8 is 22. Days from 2/15 to 8/15 is 182. Accrued interest
  - $100,000(0.055/2)(22/182) = $332.42
- Invoice price:
  - $140,000(0.9485) + $332.42 = $133,122.42

Intermediate and Long-Term Interest Rate Arbitrage (3 of 20)

- On the day of delivery, Thursday, March 8, the short invoices the long $133,122.42. The long pays for and receives the bond on that day.
- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract
  - Recall the option to deliver any T-bond with at least 15 years to maturity or first call.
  - Example: Delivery on March YY contract of 6 1/4s of May 15, YY+18.
  - Cost of delivering bond
    - $f_0(T)(CF) + AI_T - [(B + AI_t)(1 + r)^{(T-t)} - CI_{t,T}]$

Intermediate and Long-Term Interest Rate Arbitrage (4 of 20)

- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract (continued)
  - Example: On 9/15/YY–1, plan to deliver the 6 1/4s of 5/15/YY+18 on the March YYcontract on March 8, YY. $f_0(T)$ = 140 , CF = 1.0273, AIt = 2.0890, AI_T = 1.9574 (deliver on March 8), B = 148.53125. 114 days between September 15 and March 8. Reinvestment rate = 1.0%.
  - Invoice price
    - 140(1.0273) + 1.9574 = 145.7794

Intermediate and Long-Term Interest Rate Arbitrage (5 of 20)

- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract (continued)
  - There is one interim coupon paid
  - Forward price of deliverable bond
    - $$(148.53125 + 2.0890)(1.01)^{114/365} - 3.125(1.01)^{114/365} = 148.2058$$
  - So the bond would cost 2.43 (=148.2058 − 145.7794) more than it would return.

Intermediate and Long-Term Interest Rate Arbitrage (6 of 20)

- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract (continued)
  - All we can do, however, is compare this result with that for another bond. For the 6 1/8ths of November 15, YY+15 with CF = 1.0125 and price of 144 5/32, we have accrued interest of 2.0472 on September 15, 2011 and 1.9183 on March 8, YY. Coupon of 3.0625 on November 15 is reinvested at 1.0% for 114 days and grows to $3.06259(1.01)^{114/365} = 3.0720$.

Intermediate and Long-Term Interest Rate Arbitrage (7 of 20)

- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract (continued)
  - Forward price is, therefore,
    - (144.15625 + 2.0472)(1.01)175/365 – 3.0720 = 143.83
  - Invoice price is
    - 142(1.0125) + 1.9183 = 145.69.
  - Thus, this bond would produce 1.86 (= 145.69 – 143.83). So the 6 1/8, 11/YY+15 bond is better than the 6 1/4, 5/YY+18 bond.
  - See CheapestToDeliver10e.xlsm for similar calculations.

Intermediate and Long-Term Interest Rate Arbitrage (8 of 20)

- Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract (continued)
  - Why identifying the cheapest-to-deliver bond is important:
    - Identifying the true spot price
    - Calculating the correct hedge ratio

Intermediate and Long-Term Interest Rate Arbitrage (9 of 20)

- Delivery Options
  - The Wild Card Option
    - Futures market closes at 3:00 pm while spot market stays open until at least 5:00 pm.
    - This allows the holder of a short futures contract during the delivery month to potentially profit from a decline in the price of a deliverable bond during that two hour period in the expiration month.
    - Illustration: f_3 = futures price at 3:00 pm, B_3 = spot price at 3:00 pm. CF = conversion factor

Intermediate and Long-Term Interest Rate Arbitrage (10 of 20)

- Delivery Options (continued)
  - The Wild Card Option (continued)
    - Let the short own 1/CF bonds (CF must be > 1.0, so coupon must be > 6 percent). This is less than one bond per contract so additional bonds, called “the tail,” will have to be purchased in order to make delivery.
    - At 5:00 pm, the spot price is B_5. It is profitable to purchase these bonds at 5:00 pm if $B_5 < f_3(CF)$.
    - This holds because the invoice price is locked in but the spot price of the bonds can potentially fall.

Intermediate and Long-Term Interest Rate Arbitrage (11 of 20)

- Delivery Options (continued)
  - The Wild Card Option (continued)
    - If the spot price does not fall sufficiently, then the short simply waits until the next day. By the last eligible delivery day, the short would have to make delivery.
    - This is a potentially valuable option granted by the long to the short and its value would have to be reflected in a lower futures price at 3:00 pm.

Intermediate and Long-Term Interest Rate Arbitrage (12 of 20)

- Delivery Options (continued)
  - The Quality Option
    - Also called the switching option, it gives the short the right to change deliverable bonds if another becomes more attractive. This right also exists in various other futures markets.
    - Similar to this is the location option, which is the right to choose from among several eligible delivery locations. This can be valuable when the underlying is a storable commodity.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (13 of 20)

- Delivery Options (continued)
  - The End-of-the-Month Option
    - The right to make delivery any of the business days at the end of the month after the futures contract has stopped trading, around the third week of the month.
    - Similar to the wild card option because the invoice price is locked in when the futures stops trading.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (14 of 20)

- Delivery Options (continued)
  - The Timing Option
    - The right to deliver on any eligible day of the delivery month.
    - Delivery will be made early in the month if the bond earns a coupon that is less than the cost of financing.
    - Delivery will be made late in the month if the bond earns a coupon that exceeds the cost of financing.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (15 of 20)

- Implied Repo/Cost of Carry
  - Buy spot T-bond, sell futures.
  - This will produce a return (implied repo rate) of

$$\hat{r} = \left[\frac{(CF)f_0(T) + AI_T + CI_{0,T}}{B_0 + AI_0}\right]^{(1/T)} - 1$$

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (16 of 20)

- Implied Repo/Cost of Carry (continued)
  - Example: On September 15, YY–1, CTD bond on March contract is 6 3/8s maturing on August 15, YY+15. Spot price is 147 1/4, accrued interest is 0.5370, CF = 1.0370 and futures price is 142. From September 15 to March 8 is 175 days so T = 175/365 = 0.47945. There one coupon payment made with a future value of 3.1894 $((6.375/2)(1.01)^{(22/365)}$. Accrued interest on March 8th, $AI_T = 0.3853$

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (17 of 20)

- Implied Repo/Cost of Carry (continued)
  - Implied repo rate is, therefore,

$$\hat{r} = \left[\frac{140(1.0370) + 0.3853 + 3.1894}{147.5625 + 0.5370}\right]^{1/0.47945} - 1 = 0.00925$$

  - If the bond can be financed in the repo market for less than this rate, then the arbitrage would be profitable. Obviously that is not the case here.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (18 of 20)

- Treasury Bond Spread/Implied Repo Rate
  - Let time t be expiration of nearby futures and T be expiration of deferred futures.
  - Go long the nearby and short the deferred.
  - When nearby expires, take delivery and hold until expiration of deferred. This creates a forward transaction beginning at t and ending at T

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (19 of 20)

- Treasury Bond Spread/Implied Repo Rate (continued)
  - Implied repo rate

$$\hat{r} = \left[\frac{(CF(T))f_0(T) + AI_T + CI_{t,T}}{(CF(t))f_0(t) + AI_t}\right]^{1/(T-t)} - 1$$

  - Example: On September 15, YY–1 CTD was 63/8s maturing in 8/YY+14. Examine the March-June spread. March priced at $f_0(t)$ = 142 with CF(t) = 1.0370. June priced at $f_0(T)$ = 141 with CF(T) = 1.0368. AI_t (March 8) = 0.3853 and AI_T (June 8) = 1.9966. No coupons in the interim so $CI_{t,T}$ = 0. From March 8 to June 8 is 92 days.

CENGAGE

Intermediate and Long-Term Interest Rate Arbitrage (20 of 20)

- Treasury Bond Spread/Implied Repo Rate (continued)
  - Implied repo rate

$$\hat{r} = \left[\frac{1.0368(141) + 1.9966}{1.0370(142) + 0.3853}\right]^{365/92} - 1 = 0.0148$$

  - Compare to actual repo rate and note that this is a forward rate.
  - Note the turtle trade: Implied repo rate on T-bond spread to Fed funds futures rate

Stock Index Arbitrage (1 of 3)

- Stock Index Arbitrage
  - Recall the stock index futures pricing model

$$f_0(T) = S_0 e^{(r_c - \delta_c)T}$$

  - Example: Let S&P 500 = 1305.00, risk-free rate is 5.2%, dividend yield is 3% and time to expiration is 40 days so T = 40/365 = 0.1096. Futures should be at
    - $1305e^{(0.052-0.03)(0.1096)} = 1308.15$
  - Now let the actual futures price be 1309.66. This is too high so sell the futures and buy the index. Hold until expiration. Sell the stocks and buy back the futures.

Stock Index Arbitrage (2 of 3)

- Stock Index Arbitrage (continued)
  - Now find the implied repo rate. Let $f_0(T)$ be the actual futures price. Then

$$\hat{r} = \frac{\ln(f_0(T)/S_0)}{T} + \delta_c$$

  - In our example, this is

$$\hat{r} = \frac{\ln(1309.66/1305)}{0.1096} + 0.03 = 0.0625$$

  - So if you could get financing at less than this rate, the arbitrage would be worth doing.

Stock Index Arbitrage (3 of 3)

- Stock Index Arbitrage (continued)
  - Some practical considerations
    - buying and selling all stocks simultaneously
    - buying fractional contracts
    - transaction costs of about 0.005 % of spot value.
  - Program trading.
  - See Table 9.3 on 40th slide for stock index arbitrage example.

Foreign Exchange Arbitrage (1 of 2)

- Foreign exchange futures pricing model (annual compounding)

$$f_0(T) = S_0(1+r)^T / (1+\rho)^T$$

- Foreign exchange futures pricing model (continuous compounding)

$$f_0(T) = S_0 e^{(r_c - \rho_c)T}$$

Foreign Exchange Arbitrage (2 of 2)

- Example: Let spot rate for dollars = 0.7908 euro, U. S. risk-free rate is 5.84%, euro risk-free rate is 3.59% and time to expiration is 90 days so T = 90/365 = 0.2466. Futures should be priced at
  - $0.7908e^{(0.0584 - 0.0359)(0.2466)} = 0.7866$ euro
- Now let the actual futures price be 0.80 euro. This is too high so sell the futures and buy the index. Hold until expiration. Sell the euro and buy back the futures for an arbitrage profit.

Summary

Appendix 10: Determining the CBOT Treasury Bond Conversion Factor (1 of 3)

- Determine maturity in years (YRS), months (MOS) and days as of first date of expiration month. Use first call date if callable. Ignore days. Let c be coupon rate. Round months down to 0, 3, 6, or 9. Call this MOS*.
  - If MOS* = 0,

$$CF_0 = \frac{c}{2}\left[\frac{1-(1.03)^{-2*YRS}}{.03}\right] + (1.03)^{-2*YRS}$$

Appendix 10: Determining the CBOT Treasury Bond Conversion Factor (2 of 3)

- If MOS* = 3,

$$CF_3 = (CF_0 + c/2)(1.03)^{-0.5} - c/4$$

- If MOS* = 6,

$$CF_6 = \frac{c}{2}\left[\frac{1-(1.03)^{-(2*YRS+1)}}{.03}\right] + (1.03)^{-(2*YRS+1)}$$

- If MOS* = 9,

$$CF_9 = (CF_6 + c / 2)(1.03)^{-0.5} - c/4$$

CENGAGE

Appendix 10: Determining the CBOT Treasury Bond Conversion Factor (3 of 3)

- Example: 5 1/4s of February 15, YY+17 delivered on March YY contract. On March 1, YY remaining life is 16 years, 11 months, 14 days. YRS = 16, MOS = 11. Round down so that MOS* = 9. Find CF_6:

$$CF_6 = \frac{0.0525}{2}\left[\frac{1 - (1.03)^{-(2(16)+1)}}{0.03}\right] + (1.03)^{-(2(16)+1)} = 0.922128$$

- Then CF_9 is

$$CF_9 = (0.922128 + 0.0525 / 2)(1.03)^{-0.5} - 0.0525 / 4 = 0.9213$$

CENGAGE

TABLE 9.1 FEDERAL FUNDS CARRY ARBITRAGE (1 of 2)

Scenario: On March 1, the one-month LIBOR rate is 5.50 percent and the two-month LIBOR rate is 6.00 percent. The April fed funds futures is quoted at 93.75. An arbitrage opportunity is available. Contract size is $5,000,000, and this example illustrates the arbitrage strategy for one contract.

DATE	SPOT MARKET	FUTURES MARKET
March 1	Borrow present value of $5,000,000 at one-month LIBOR ($4,977,187.89); lend same amount at two-month LIBOR.	Sell one April fed funds futures contract at 93.75.
April 1	Repay borrowing (−$5,000,000). Pay off present value of loan.	Buy one April fed funds futures contract to offset initial futures position.

Analysis: The key insight is that the payoff required on April 1 of the two-month LIBOR loan depends on the one-month LIBOR rate observed on April 1. Also, we assume no basis risk between one-month LIBOR rates and fed funds futures implied rates as long as we are not in the delivery month. The loan balance due on May 1 is

(To continued)

CENGAGE

TABLE 9.1 FEDERAL FUNDS CARRY ARBITRAGE (2 of 2)

$$LB_2 = LB_0\left[1 + r_2\left(\frac{60}{360}\right)\right] = \$4{,}977{,}187.89\left[1 + 0.06\left(\frac{60}{360}\right)\right] = \$5{,}026{,}959.77.$$

Assuming that the one-month LIBOR rate observed on April 1 is 7.25 percent, then the present value of the initial two-month loan after one month is

$$PV_1(LB_2) = \frac{LB_2}{\left[1 + r_{1,1}\left(\frac{30}{360}\right)\right]} = \frac{\$5{,}026{,}959.77}{\left[1 + 0.0725\left(\frac{30}{360}\right)\right]} = \$4{,}996{,}770.94$$

Assuming no basis risk, the fed funds futures price in one month is 92.75 =(100 − 7.25.) Therefore, the gain on the fed funds futures contract is $4,166.67[=(93.75−92.75)×($5,000,000)×(30/360)/100]. Adding the gain on the futures contract with the proceeds from the present value of the two-month loan, we have $5,000,937.61. Paying off the one-month loan provides arbitrage profits of $937.61. The implied one-month repo rate, therefore, is

$$\left(\frac{\$5{,}000{,}937.61}{\$4{,}977{,}187.89} - 1\right)\frac{360}{30} = 0.0573,$$

or 5.73 percent. The one-month LIBOR rate is 5.50 percent, indicating the potential arbitrage opportunity.

(Return to previous slide) (Return to text slide)

CENGAGE

TABLE 9.2 EURODOLLAR ARBITRAGE (1 of 3)

Scenario: On September 16, a London bank needs to issue $10 million of 180-day Eurodollar time deposits. The current rate on such time deposits is 8.75. The bank is considering the alternative of issuing a 90-day time deposit at its current rate of 8.25 and selling a Eurodollar futures contract. If the 180-day time deposit is issued, the bank will have to pay back

$$\$10{,}000{,}000\left[1+0.0875\left(180/360\right)\right]=\$10{,}437{,}500,$$

which is an effective rate of

$$\left(\frac{\$10{,}437{,}500}{\$10{,}000{,}000}\right)^{365/180}-1=0.0907.$$

The rates available in the spot and futures market are such that the bank can obtain a better rate by doing the following:

(To continued)

TABLE 9.2 EURODOLLAR ARBITRAGE (2 of 3)

DATE	SPOT MARKET	FUTURES MARKET
September 16	The 90-day time deposit rate is 8.25.	December Eurodollar IMM Index is at 91.37. Price per $100 face value: $100 - (100 - 91.37)(90/360) = 97.8425$.
	Issue 90-day time deposit for $10,000,000	Price per contract: $978,425. **Sell 10 contracts**
December 16	The 90-day time deposit matures and the bank owes $\$10{,}000{,}000[1 + 0.0825(90/360)] = \$10{,}206{,}250$.	December Eurodollar IMM Index is at 92.04. Price per $100 face value: $100 - (100 - 92.04)(90/360) = 98.01$.
	The rate on new 90-day time deposits is 7.96.	Price per contract: $980,100.
	Issue new 90-day time deposit for $10,223,000	**Buy 10 contracts**

(Return to previous slide) (To Continued)

CENGAGE

Chance/Brooks, An Introduction to Derivatives and Risk Management, 10th Edition. © 2016 Cengage. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

TABLE 9.2 EURODOLLAR ARBITRAGE (3 of 3)

Analysis: On September 16, the bank received $10,000,000 from the newly issued 90-day time deposit. On December 16, it bought back its 10 futures contracts at a loss of 10($980,100−$978,425)=$16,750. It issued $10,223,000 of new time deposits using $16,750 to cover the loss in the futures account and the remaining $10,206,250 to pay off the maturing 90-day time deposit. On March 16, it paid off the new time deposit, owing

$$\$10,223,000[1+0.0769(90/360)] = \$10,426,438.$$

Thus, on September 16, it received $10,000,000, and on March 16, it paid out $10,426,438. It had no other cash flows in the interim. Thus, its effective borrowing cost for 180 days was

$$\left(\frac{\$10,426,438}{\$10,000,000}\right)^{365/180} - 1 = 0.0884.$$

which is 23 basis points less than the cost of a 180-day Eurodollar time deposit.

(Return to previous slide) (Return to text slide)

TABLE 9.3: STOCK INDEX ARBITRAGE (1 of 2)

Scenario: On November 8, the S&P 500 index is at 1,305, the continuously compounded dividend yield is 3 percent, and the continuously compounded risk-free rate is 5.2 percent. The December futures contract, which expires in 40 days, is priced at 1,316.30. Its theoretical price is

$$f = 1{,}305e^{(0.052-0.03)(0.1096)} = 1{,}308.15,$$

where T = 40/365 = 0:1096. Thus, the futures contract is overpriced and the carry arbitrage transaction will be executed using $20 million. Transaction costs are 0.5 percent of the dollars invested.

DATE	SPOT MARKET	FUTURES MARKET
November 8	The S&P 500 is at 1,305. The stocks have a dividend yield of 3 percent. The risk-free rate is 5.2 percent. **Buy $20 million of stock in the same proportions as make up the S&P 500**	The S&P 500 futures, expiring on December 18, is at 1,316.30. The appropriate number of futures is $20 million/[(1,305)(250)] = 61.30.* **Sell 61 contracts**
December 18	The S&P 500 is at 1,300.36. The stocks will be worth (1,300.36/1305)($20 million) = $19.928889 million. The $20 million invested in the stocks effectively costs ($20 million)$e^{(0.052-0.03)(0.1096)}$ = $20.048242 million. Transactions costs are $20 million (0 005) = $100,000 (includes futures costs). **Sell stocks**	Futures expires at the S&P 500 price of 1,300.36. **Close out futures at expiration**

(To continued)

TABLE 9.3: STOCK INDEX ARBITRAGE (2 of 2)

Analysis

Profit on stocks

$$\begin{array}{rl} \$19{,}928{,}889 & \text{(received from sale of stocks)} \\ -\$20{,}048{,}282 & \text{(invested in stocks)} \\ \hline \$119{,}393 & \end{array}$$

Profit on futures

$$\begin{array}{rl} 61(250)(1{,}316.30) & \text{(sale price of futures)} \\ -61(250)(1{,}300.36) & \text{(purchase price of futures)} \\ \hline \$243{,}085 & \end{array}$$

Overall profit

$$\begin{array}{rl} \$243{,}085 & \text{(from futures)} \\ -\$119{,}393 & \text{(from stock)} \\ -\$100{,}000 & \text{(transaction costs)} \\ \hline \$23{,}692 & \end{array}$$

*The appropriate number of futures contracts to match $20 million of stock is $20 million divided by the index price, not the futures price, times the multiplier. Even though the $20 million is allocated across 500 different stocks, it is equivalent to buying $20 million/1,305 = 15,326 "shares" of the S&P 500. The appropriate number of futures is one for each equivalent "share" of the S&P 500. Each futures, of course, has a $250 multiplier.

(Return to previous slide) (Return to text slide)